

常用対数値の分数近似

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北海道からの発信です。全国で高等学校数学教育に関わっていらっしゃる先生やOBで常用対数に興味をもった方々、大学や研究機関におかれて数値解析をしていらっしゃる方々へ発信します。また、これから記述する事柄について、スーパーコンピュータやパソコンで検証していただける方がおられましたら、よろしくお願い申し上げます。

常用対数値 $\log_{10}2$ と $\log_{10}3$ を分数近似することができないかと思い立ち、常用対数値の小数部分を連鎖させることで分母が24桁までの近似分数を多数入手ことができました。25桁以上については、どなたかの解明を待ちたいと思います。以下に、分母が1桁、2桁、3桁、4桁、5桁、……の分数を記述していきます。後述で2及び3の累乗展開を行ったときの、先行する数字群の一部を記述しますので、この部分についてのご検討、ご検証よろしくお願いたします。

まず、 $\log_{10}2$ についての近似分数群（分母の桁数において複数あります。 $\log_{10}3$ でも同様です）のうち最接近して近似しているものを記述していきます。記述する分数は、すべてユークリッドの互除法により分子、分母が互いに素な状態にあります。

$$\frac{2}{7}, \frac{1}{3}, \frac{3}{10}, \frac{205}{681}, \frac{146}{485}, \frac{643}{2,136}, \frac{21,306}{70,777}, \frac{12,655}{42,039}, \frac{97,879}{325,147}, \frac{76,573}{254,370}, \frac{1,936,274}{6,432,163}, \frac{1,838,395}{6,107,016}$$

$$\frac{15,392,313}{51,132,157}, \frac{13,456,039}{44,699,994}, \frac{267,380,264}{888,218,011}, \frac{103,873,643}{345,060,773}, \frac{2,791,132,322}{9,271,940,877}, \frac{579,001,193}{1,923,400,330}, \frac{24,793,177,656}{82,361,153,417}$$

$$\frac{174,131,244,785}{578,451,474,249}, \frac{149,338,067,129}{496,090,320,832}, \frac{1,865,857,337,322}{6,198,243,909,905}, \frac{845,863,046,269}{2,809,896,217,828}, \frac{23,062,019,849,360}{76,610,371,662,439}, \frac{169,743,431,341,081}{563,875,473,494,521}$$

$$\frac{655,911,705,514,963}{2,178,891,522,315,645}, \frac{2,793,390,253,400,937}{9,279,441,562,757,101}, \frac{165,736,237,459,304,329}{550,563,863,556,986,329}, \frac{764,456,273,488,149,454}{2,539,468,772,213,180,761}$$

$$\frac{564,882,928,145,201,079}{1,876,500,469,327,782,617}, \frac{11,199,596,541,212,005,342}{37,204,254,401,655,489,641}, \frac{9,870,257,339,578,654,810}{32,788,285,160,114,526,263}, \frac{186,405,123,595,704,039,231}{619,224,417,103,520,433,763}$$

$$\frac{120,536,883,550,065,357,707}{400,414,859,935,128,459,295}, \frac{1,830,452,446,333,404,376,290}{6,080,631,407,830,237,868,707}, \frac{1,402,973,555,637,569,621,645}{4,660,577,270,856,460,516,354}$$

$$\frac{24,343,897,576,580,157,004,135}{80,868,677,298,701,998,104,839}, \frac{15,619,114,235,608,969,877,327}{51,885,574,396,524,586,113,657}, \frac{118,058,582,990,233,976,268,097}{392,182,123,677,849,514,786,781}$$

2の累乗を展開した場合の先行数字群、桁数を記述します。

指数	先行数字群	桁数
3	8	1
7	128	3
10	1024	4
681	100,329,130,...	206
485	998,95...	146
2,136	100,016,289,...	644
70,777	100,000,716,...	21,307
42,039	999,971,7...	12,655
325,147	100,000,036,...	97,880
254,370	999,993,200,...	76,573
6,432,163	100,000,004,668,...	1,936,275
6,107,016	999,999,686,3...	1,838,395
51,132,157	100,000,001,316,7...	15,392,314
44,699,994	999,999,966,...	13,456,039
345,060,773	999,999,998,797,...	103,873,643
888,218,011	100,000,000,357,...	267,380,265
9,271,940,877	100,000,000,106,283...	2,791,132,323
1,923,400,330	999,999,999,972,1...	579,001,193
82,361,153,417	100,000,000,000,408,7...	24,793,177,657
578,451,474,249	100,000,000,000,075,136,...	174,131,244,786
496,090,320,832	999,999,999,996,663,78...	149,338,067,129
6,198,243,909,905	100,000,000,000,008,982...	1,865,857,337,323
2,809,896,217,828	999,999,999,999,669,231...	845,863,046,269
76,610,371,662,439	999,999,999,999,995,780,128,...	23,062,019,849,360
487,265,101,832,082	100,000,000,000,000,321,613,...	146,681,411,491,721
563,875,473,494,521	999,999,999,999,998,996,261,...	169,743,431,341,081
2,178,891,522,315,645	100,000,000,000,000,020,491,...	655,911,705,514,964
550,563,863,556,986,329	999,999,999,999,999,997,400,772,...	165,736,237,459,304,329
2,539,468,772,213,180,761	100,000,000,000,000,000,557,788...	764,456,273,488,149,455

1,876,500,469,327,782,617	999,999,999,999,999,999,505,450,048,...	564,882,928,145,201,079
37,204,254,401,655,489,641	100,000,000,000,000,000,001,136,079,...	11,199,596,541,212,005,343
32,788,285,160,114,526,263	999,999,999,999,999,999,948,121,955,...	9,870,257,339,578,654,810
619,224,417,103,520,433,763	100,000,000,000,000,000,000,341,704,...	186,405,123,595,704,039,232
400,414,859,935,128,459,295	999,999,999,999,999,999,998,491,098,...	120,536,883,550,065,357,707
6,080,631,407,830,237,868,707	100,000,000,000,000,000,000,008,808,127,...	1,830,452,446,333,404,376,291
4,660,577,270,856,460,516,354	999,999,999,999,999,999,999,688,835,68,...	1,402,973,555,637,569,621,645
80,868,677,298,701,998,104,839	100,000,000,000,000,000,000,003,540,102,...	24,343,897,576,580,157,004,136
51,885,574,396,524,586,113,657	999,999,999,999,999,999,999,994,240,257,...	15,619,114,235,608,969,877,327
392,182,123,677,849,514,786,781	100,000,000,000,000,000,000,000,084,...	118,058,582,990,233,976,268,097

続いて、 $\log_{10}3$ についての近似分数を記述し、後述で先行数字群及び桁数を記述していきます。

$$\frac{1}{2}, \frac{5}{9}, \frac{31}{65}, \frac{21}{44}, \frac{417}{874}, \frac{73}{153}, \frac{4,223}{8,851}, \frac{1,074}{2,251}, \frac{8,519}{17,855}, \frac{154,416}{323,641}, \frac{317,351}{665,137}, \frac{4,108,525}{8,611,071}, \frac{3,002,056}{6,292,019}, \frac{10,112,637}{21,195,109}, \frac{43,452,604}{91,072,455}$$

$$\frac{441,636,621}{925,627,640}, \frac{97,017,845}{203,340,019}, \frac{4,332,463,058}{9,080,423,509}, \frac{2,920,647,987}{6,121,395,679}, \frac{45,221,534,876}{94,779,963,015}, \frac{395,602,275,471}{829,144,104,476}, \frac{132,840,974,486}{278,421,833,385}, \frac{4,496,148,596,615}{9,423,492,565,337}$$

$$\frac{661,284,224,443}{1,385,987,771,246}, \frac{30,020,551,400,920}{62,920,171,977,161}, \frac{432,719,278,902,811}{906,937,753,500,543}, \frac{311,975,789,074,688}{653,871,077,820,653}, \frac{4,488,404,536,873,755}{9,407,261,765,169,032}$$

$$\frac{9,791,992,951,143,451}{20,523,070,088,120,133}, \frac{19,272,010,113,212,214}{40,392,269,098,419,613}, \frac{154,488,395,602,275,471}{323,792,023,865,177,557}, \frac{2,211,168,395,602,275,471}{4,634,395,942,397,445,157}$$

$$\frac{16,163,741,132,840,974,486}{33,877,639,569,626,205,299}, \frac{34,538,652,395,602,275,471}{72,389,675,081,649,855,755}, \frac{257,934,310,124,338,771,043}{540,605,365,141,175,195,584}, \frac{223,395,657,520,633,309,472}{468,215,690,059,525,339,829}$$

$$\frac{2,045,099,570,132,840,974,486}{4,286,330,885,617,377,914,216}, \frac{3,866,803,483,395,602,275,471}{8,104,446,081,175,230,488,603}, \frac{5,911,903,053,347,582,430,985}{12,390,776,966,792,608,402,819}$$

3の累乗展開した場合の先行数字群、桁数を記述します。

指数(累乗)	先行数字群	桁数
2	9	1
9	19,683	5
65	10,301,051,...	32
44	984,770,902,...	21
874	100,919,856,...	418
153	998,968,909,594,...	73
8,851	100,051,942,243,...	4,224
2,251	999,871,924,512,...	1,074
17,855	100,000,695,284,...	8,520
323,641	999,997,067,134,...	154,416
665,137	100,000,108,708,...	317,352
8,611,071	100,000,022,644,44...	4,108,526
6,292,019	999,999,898,062,367,...	3,002,056
21,195,109	100,000,002,256,918,819,...	10,112,638
91,072,455	999,999,988,339,114,5...	43,452,604
925,627,640	100,000,000,789,796,973,780,...	441,636,622
203,340,019	999,999,999,247,416,83...	97,017,845
9,080,423,509	999,999,999,991,692,...	4,332,463,059
6,121,395,679	100,000,000,036,383,...	2,920,647,987
94,779,963,015	100,000,000,024,753,092,292,...	45,221,534,877
829,144,104,476	100,000,000,000,325,7...	395,602,275,472
278,421,833,385	999,999,999,998,31...	132,840,974,486
9,423,492,565,337	100,000,000,000,091,910,111,...	4,496,148,596,616
1,385,987,771,246	999,999,999,999,890,834,...	661,284,224,443
62,920,171,977,161	100,000,000,000,002,816,078,343,...	30,020,551,400,921
906,937,753,500,543	100,000,000,000,000,336,36...	432,719,278,902,812
653,871,077,820,653	999,999,999,999,999,886,22...	311,975,789,074,688
9,407,261,765,169,032	100,000,000,000,000,188,454,028,...	4,488,404,536,873,756
20,523,070,088,120,133	999,999,999,999,999,950,315,596,...	9,791,992,951,143,451

40,392,269,098,419,613	100,000,000,000,000,001,440,911,...	19,272,010,113,212,215
323,792,023,865,177,557	100,000,000,000,000,000,149,500,690,...	154,488,395,602,275,472
4,634,395,942,397,445,157	100,000,000,000,000,000,006,392,572,...	2,211,168,395,602,275,472
33,877,639,569,626,205,299	999,999,999,999,999,970,453,00...	16,163,741,132,840,974,486
72,389,675,081,649,855,755	100,000,000,000,000,000,000,483,174,...	34,538,652,395,602,275,472
540,605,365,141,175,195,584	100,000,000,000,000,000,000,427,520,432,...	257,934,310,124,338,771,044
468,215,690,059,525,339,829	999,999,999,999,999,999,443,462,145,...	223,395,657,520,633,309,472
12,390,776,966,792,608,402,819	100,000,000,000,000,000,000,002,52...	5,911,903,053,347,582,430,986