83th Meeting of the Practice of Mathematics Education Study Group[*] ～第８３回数学教育実践研究会

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1 Let’s MATH！～英語で数学をしよう！

1.1 はじめに

英語で書かれた数学の問題や科学論文を高校の段階で取り入れることは、英語の必要性が高まるグローバルな社会で外国の文献を読まざるを得ない次世代の若者にとって大きな価値がある。平易な英文による表記で高校レベルの数学問題を授業でプリントや課題として生徒に実践練習させることで普段の授業とは別の視野で数学に興味、関心を深めることにつながるだろう。

また、数学教師として教科指導のスキルに加え、新たな言語で数学の問題を考え、解かせることで世界の数学について、知識や理解を深め、生徒とのコミュニケーションが期待できる。今回は”The Basics :Twenty-Seven Ploblems” [1]より問題を取り上げる。

1.2 ploblems

1. The measure of an angle is 3 times the measure of its complement. Find the measure of the angle in degrees.

2. Notice that the product of the digits of 124 is 8. For many other three-digit positive integers is the product of the digits equal to 8?

3. Steven takes his favorite number and adds 7, multiplies the resulting number by 6, squares the resulting number, and divides by 9. The final result of these operations is 16. Given that Steven’s favorite number is not 5, what is his favorite number?

4. Given that a, b, and c are all positive, and that $ab = 21$, $ac = 18$, and $bc = 42$, find $a + b + c$.

5. If $\frac{x}{4y} = \frac{y}{4z}$, find the value of $3y^4 - 3(xz)^2 + 3$.

6. Given the sequence $2, 4, 6, 10, 16, 26, \ldots$, find the 12th term.

7. Bill will be x years old in the year x^2. If Bill was born between 1900 and 2000, in what year was he born?

8. Five fair coins are tossed. What is the probability that exactly two heads and three tails turn up? Express your answer as a decimal.

9. Bob is throwing turtles at a dartboard with two regions. If he hits the smaller region, he gets 15 points; if he hits the larger region, he gets 7 points. What is the largest score that Bob cannot get?

10. A tennis tournament starts out with 140 players. Each game is played between two players; at the end of the game, the winner advances and the loser is knocked out. There are no ties. If the tournament has a minimum number of byes, how many games must be played to determine a single winner?

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11. How many positive integers less than or equal to 1000 are multiples of 2 or 3?

12. Find $x^2+y^2+z^2$ if

$$x+y+z=3$$
$$2x-y+z=5$$
$$3x+2y-z=16$$

13. If $x+\dfrac{1}{x}=8$, find the value of $x^4+\dfrac{1}{x^4}$.

14. If the prime factorization of positive integer N is $2^4\cdot 3^9\cdot 11^{121}$, then how many positive integer factors does N have?

15. If $2^{16x^2-4x+7}=16^{x^2+2x+1}$, what is x?

1.3 Ploblems &解　答

1. The measure of an angle is 3 times the measure of its complement. Find the measure of the angle in degrees.

   解答:
   求める角度を x 度、その余角を y 度とすると, $x+y=90$. 求める角度は余角の 3 倍だから $x=3y$. 代入して $3y+y=90$, $4y=90$、 $y=22.5$. これより, $x=3y=3\cdot 22.5=\mathbf{67.5}$.

2. Notice that the product of the digits of 124 is 8. For many other three-digit positive integers is the product of the digits equal to 8?

   解答:
   ３個の数字を掛けて８になるのは
   $1\cdot 1\cdot 8=8$ から $(1,1,8)$:$118,181,811$, $1\cdot 2\cdot 4=8$ から $(1,2,4)$: $124,142,214,241,412,421$, $2\cdot 2\cdot 2=8$ から $(2,2,2)$: 222
   以上の 10 個ある。124 以外に **9** 個ある。

3. Steven takes his favorite number and subtracts 7, multiplies the resulting number by 6, squares the resulting number, and divides by 9. The final result of these operations is 16. Given that Steven's favorite number is not 5, what is his favorite number?

   解答:
   Steven の好きな数を x とすると.
   $\dfrac{\{6(x-7)\}^2}{9}=16.$
   $\{6(x-7)\}^2=144$
   $6(x-7)=\pm 12$
   $x-7=2$ or $x-7=-2$
   $x=9$ or $x=5$. よって求める数は $x=\mathbf{9}$.

4. Given that a, b, and c are all positive, and that $ab=21$, $ac=18$, and $bc=42$, find $a+b+c$.

   解答:
   $ab=21$、 $ac=18$ から, $\dfrac{c}{b}=\dfrac{(ac)}{(ab)}=\dfrac{18}{21}=\dfrac{6}{7}$.
   $\dfrac{c}{b}=\dfrac{6}{7}$ と $bc=42$ を辺々掛けて, $c^2=36$, よって $c=\pm 6$. $c>0$ より $c=6$. 与式に代入して, $a=3$. $b=7c=6$ により $a+b+c=3+7+6=\mathbf{16}$.

5. If $\dfrac{x}{4y}=\dfrac{y}{4z}$, find the value of $3y^4-3(xz)^2+3$.

   解答:
   $\dfrac{x}{4y}=\dfrac{y}{4z}$ の両辺に $4yz$ 掛けて $xz=y^2$.

   よって $3y^4-3(xz)^2+3$
   $=3y^4-3(y^2)^2+3=3y^4-3y^4+3=\backslash mathbf\{3\}$.

6. Given the sequence $2,4,6,10,16,26,\ldots$, find the 12th term.

   解答:
   $2+4=6, 4+6=10$ と前の 2 項の和が 3 番目の項になるので, 7 番目の項は $16+26=42$, 8 番目の項は $26+42=68$, 9 番目の項は $42+68=110$, 10 番目の項は $68+110=178$, 11 番目の項は $110+178=288$, 12 番目の項は $178+288=\mathbf{466}$.

7. Bill will be x years old in the year x^2. If Bill was born between 1900 and 2000, in what year was he born?

解答:
Bill はx^2年にx歳になるから, x^2-x年は０歳である. $1900 < x^2-x < 2000$ から これを満たすxを求める。$x=44$のとき, $x^2-x=1892$, $x=45$のとき,$x^2-x=1980$, $x=46$のとき ,$x^2-x=2070$. Bill の生まれたのは 1900 年 から 2000 年の間だから, $x=45$ のときで生まれた年は **1980**.

8. Five fair coins are tossed. What is the probability that exactly two heads and three tails turn up? Express your answer as a decimal.

解答:
５枚の coin の出方は $2^5=32$ 通りある. ２枚が表で３枚が裏になるのは１０通りだから、その確率は $\frac{10}{32}=\mathbf{0.3125}$.

9. Bob is throwing turtles at a dartboard with two regions. If he hits the smaller region, he gets 15 points; if he hits the larger region, he gets 7 points. What is the largest score that Bob cannot get?

解答:
Bob が獲得できる得点を並べると: 7, 14, 15, 21, 22, 28, 29, 30, 35, 36, 37, 42, 43, 44, 45, 49, 50, 51, 52, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90$\cdots$
84 以降はすべての数が得点可能となるから求めるのは **83**.

10. A tennis tournament starts out with 140 players. Each game is played between two players; at the end of the game, the winner advances and the loser is knocked out. There are no ties. If the tournament has a minimum number of byes, how many games must be played to determine a single winner?

解答:
140 人の選手が試合に参加すると 139 人が負ける。一試合で一人の選手が負けるから **139** 試合することになる.

11. How many positive integers less than or equal to 1000 are multiples of 2 or 3?

解答:
２の倍数は $2,4,\ldots 1000$ の 500 個. ３の倍数は $3,6,\ldots 999$ の 333 個. ２と３の共通公倍数６の倍数、$6,12,18,\ldots 996$ の 166 個がそれぞれに含まれているから $500+333-166=\mathbf{667}$ 個

12. Find $x^2+y^2+z^2$ if

$$x+y+z=3$$
$$2x-y+z=5$$
$$3x+2y-z=16$$

解答:
1 式＋ 2 式より $3x+2z=8$
2 式 $\times 2+3$ 式より $7x+z=26$.

$$14x+2z=52$$
$$3x+2z=8$$

辺々引き $11x=44$ から $x=4$. $y=1$、 $z=-2$.
よって
$x^2+y^2+z^2=4^2+1^2+(-2)^2=16+1+4=\mathbf{21}$.

13. If $x+\frac{1}{x}=8$, find the value of $x^4+\frac{1}{x^4}$

解答:
$x+\frac{1}{x}=8$ の両辺二乗して

$$\begin{aligned} x^2+2\cdot x\cdot\frac{1}{x}+\left(\frac{1}{x}\right)^2 &= 64 \\ x^2+2+\frac{1}{x^2} &= 64 \\ x^2+\frac{1}{x^2} &= 62 \end{aligned}$$

これをさらに両辺二乗して

$$\begin{aligned} (x^2)^2+2\cdot x^2\cdot\frac{1}{x^2}+\left(\frac{1}{x^2}\right)^2 &= 3844 \\ x^4+2+\frac{1}{x^4} &= 3844 \\ x^4+\frac{1}{x^4} &= \mathbf{3842} \end{aligned}$$

14. If the prime factorization of positive integer N is $2^4 \cdot 3^9 \cdot 11^{121}$, then how many positive integer factors does N have?

解答:
$a^k \cdot b^l \cdot c^m \cdot d^n \cdots$ と表された数のとき、その約数の個数は $(k+1)(l+1)(m+1)(n+1)\cdots$.

よって $2^4 \cdot 3^9 \cdot 11^{121}$ の約数の個数は
$(4+1)(9+1)(121+1) = 5 \cdot 10 \cdot 122 = \mathbf{6100}$ 個.

15. If $2^{16x^2-4x+7} = 16^{x^2+2x+1}$, what is x?

解答:

$$
\begin{aligned}
2^{16x^2-4x+7} &= (2^4)^{x^2+2x+1} \\
2^{16x^2-4x+7} &= 2^{4(x^2+2x+1)} \\
2^{16x^2-4x+7} &= 2^{4x^2+8x+4} \\
16x^2-4x+7 &= 4x^2+8x+4 \\
12x^2-12x+3 &= 0 \\
4x^2-4x+1 &= 0 \\
(2x-1)^2 &= 0 \\
x &= \mathbf{\frac{1}{2}}
\end{aligned}
$$

1.4 New Ploblems &解　答

Group the numbers as follows:
7| 14, 15 | 21,22 | 28, 29, 30 | 35, 36,37| 42, 43, 44, 45 | 49, 50,51, 52 | 56, 57, 58, 59, 60 | 63, 64, 65, 66, 67 | 70, 71, 72,73, 74, 75 | 77, 78, 79, 80,81,82|
84,85,86,87,88,89,90|91,92,93,94,95,96,97|
98,99,100,101,102,103,104, 105|
106,107,108,109,110,111,112,113|
114,115,116,117,118,119,120,121,122|
123,124,125,126,127,128,129,130,131| $\cdots$.

Notice that there is one number in the first group, two numbers in the second and the third group,three numbers in the 4th and 5th group, etc.

(1)Find the first number in the 21th group.

(2)What is the sum of the numbers in the 21th group?

(3)What the group and what the term is 243 ?

解答:
(1) 第 $2n$ 群, 第 $2n+1$ 群 $(n \leqq 6)$ の最初の数は、それぞれ $14n, 14n+7$ となる。$n \geqq 7$ のとき、第 $2n$ 群の最初の数は、$98, 114, 132, 152 \cdots$ から階差数列は $16, 18, 20 \cdots$ だ から

$$b_m = 98 + \sum_{k=1}^{m-1}(2k+14) = m^2+13m+84 (m \geqq 2)$$
これは $m=1$ のとき成り立つ。また、$m = n-6$.
第 $2n+1$ 群の最初の数は、$106, 123, 142, 163 \cdots$ となる。

階差数列は $17, 19, 21 \cdots$ から
$$c_l = 106 + \sum_{k=1}^{l-1}(2k+15) = l^2 + 14l + 91 (l \geqq 2)$$
これは $l=1$ のとき成り立つ。また、$l = n-6$.
第 21 群 は $2n+1 = 21$ より $n = 10, l = 4$ から
$c_4 = 4^2 + 14 \cdot 4 + 91 = \mathbf{163}$

(2) 項数は $n+1$ より、初項 163, 項数は 22, 公差 1 から
$$S = \frac{22(2 \cdot 163 + 21)}{2} = \mathbf{1848}$$

(3) $l = 7$ のとき、$c_7 = 7^2 + 14 \cdot 7 + 91 = 238$
$n = 13$ より 243 は第 **27** 群で **6** 番目にある。