

四分円内の正方形と正三角形の1辺について I

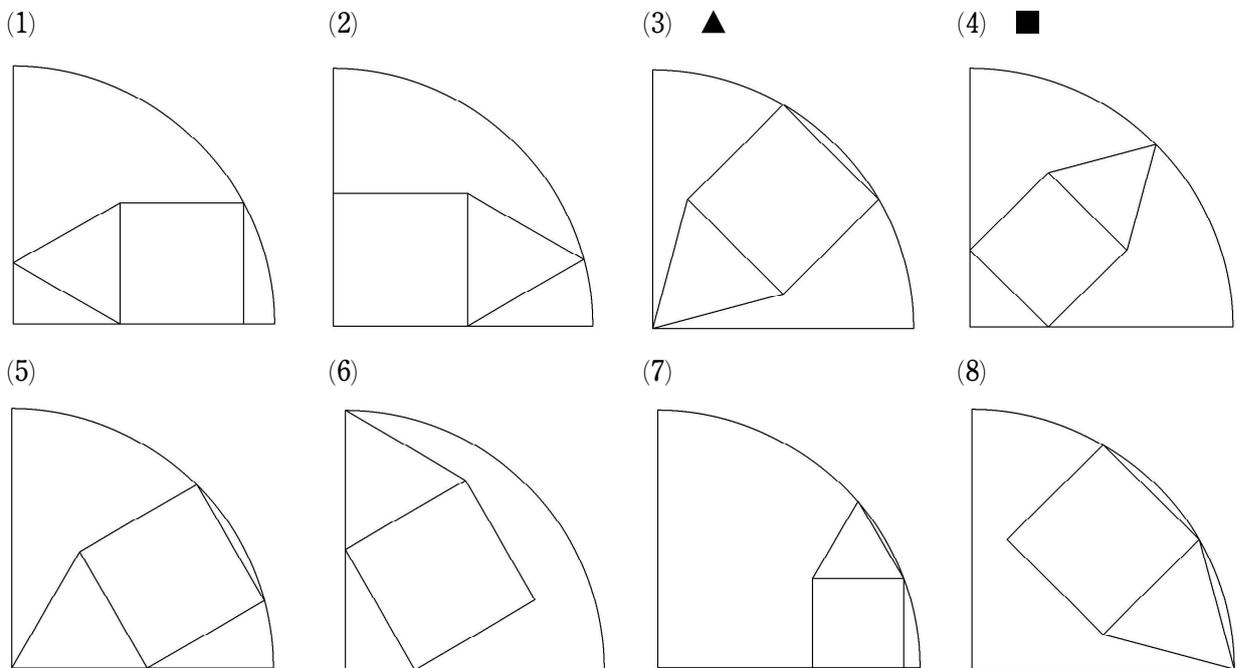
数実研会員 時岡郁夫

半径1の四分円内に正方形1個と正三角形1個をいろいろな配置で内接させる(50通り)。正方形, 正三角形の1辺の長さをそれぞれ a, b とおいて求めてみた。

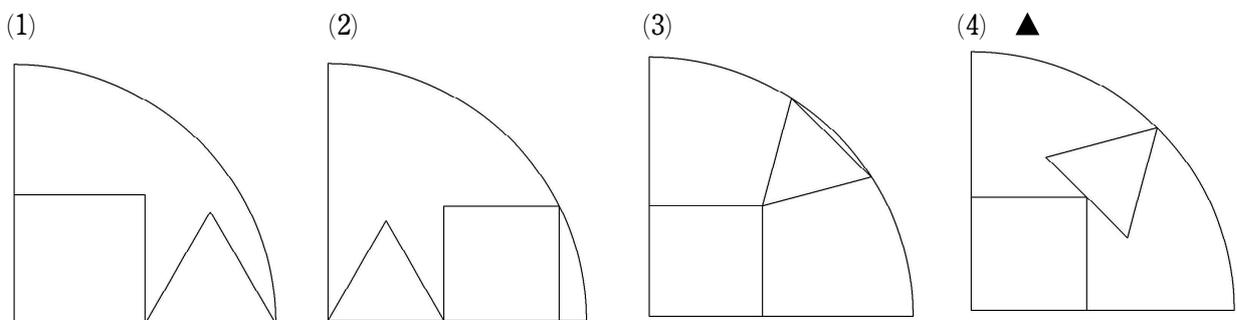
1. 正方形と正三角形の1辺が等しい ($a=b$) のとき (辺を共有する) 8通り
2. 正方形と正三角形の1辺が等しい ($a=b$) のとき (辺を共有しない) 11通り
3. 正方形の1辺が正三角形の1辺より大 ($a>b$) のとき 15通り
4. 正方形の1辺が正三角形の1辺より小 ($a<b$) のとき 16通り

ただし, ■印の問題の正方形と▲印の問題の正三角形は, 四分円の中心角の二等分線に関して対称, ▼印の問題の図形は, 四分円の中心角の三等分線(正三角形の角の二等分線)に関して対称である。また, ★印の問題は, a または b の値が複雑になるか或いは代数的に求めることができないことが分かった。

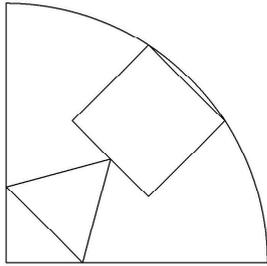
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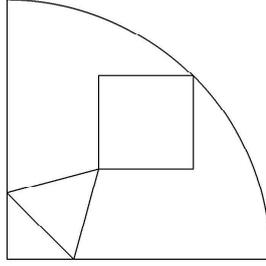
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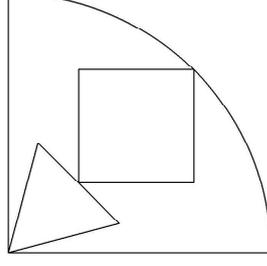
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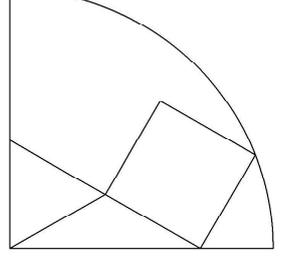
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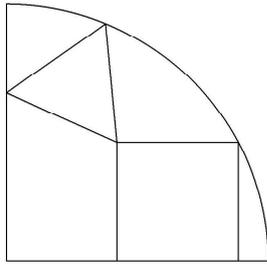
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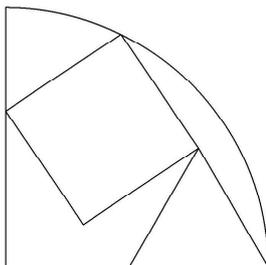
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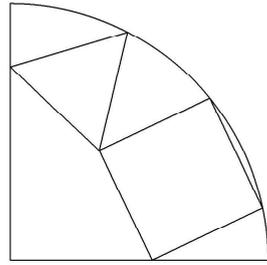
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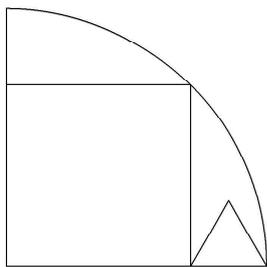


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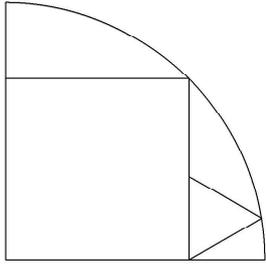


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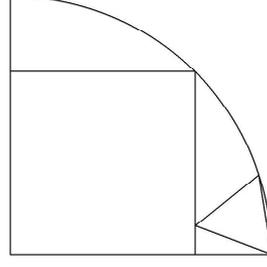
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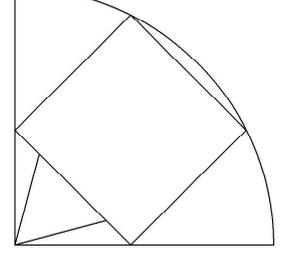
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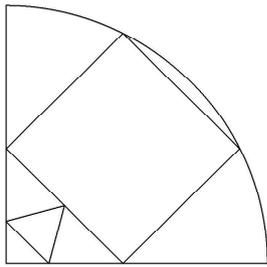
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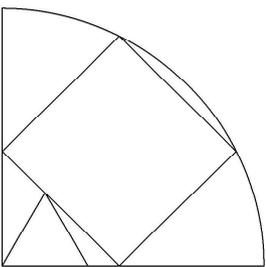
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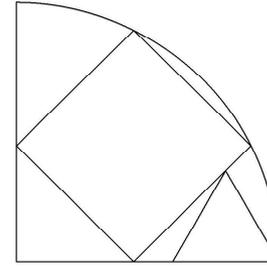
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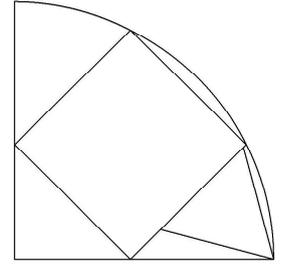
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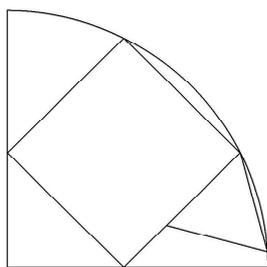
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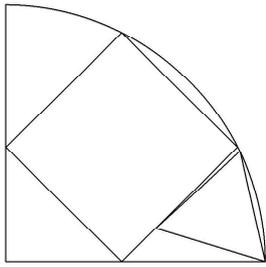
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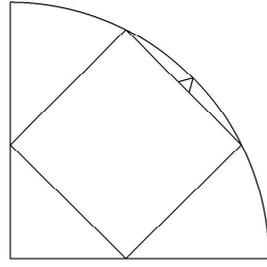
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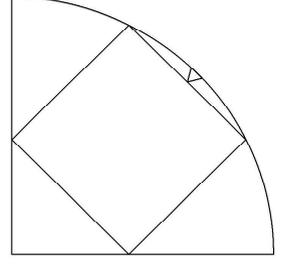
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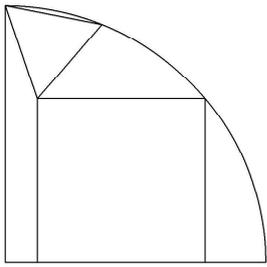
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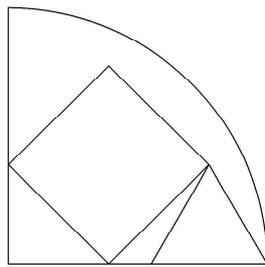
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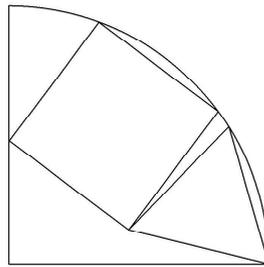
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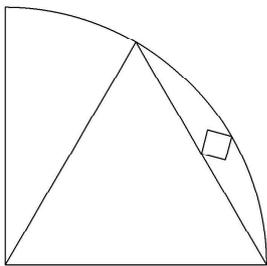


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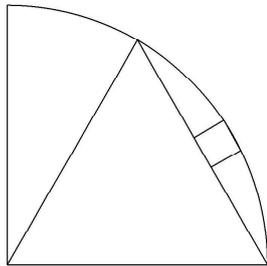


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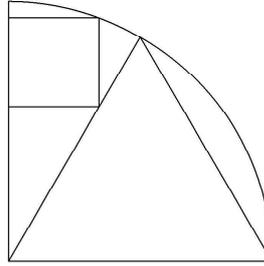
(1) ▼



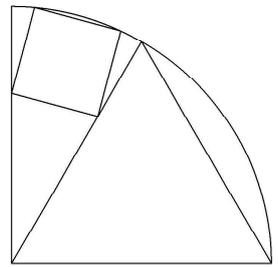
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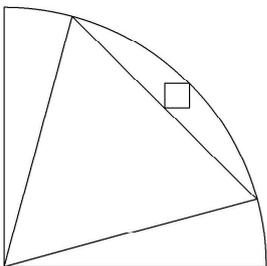
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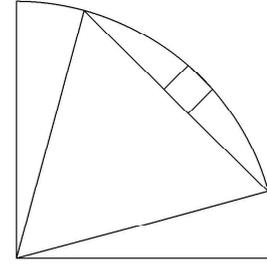
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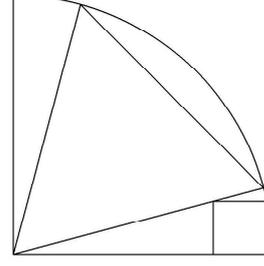
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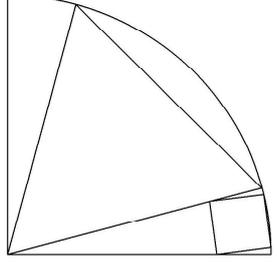
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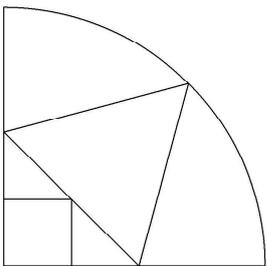
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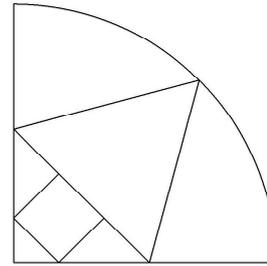
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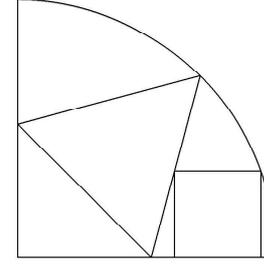
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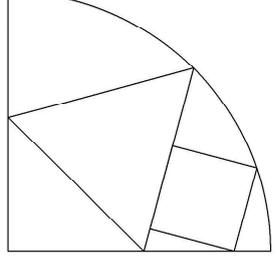
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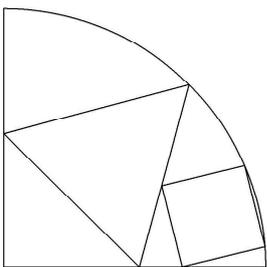
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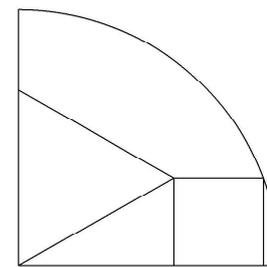
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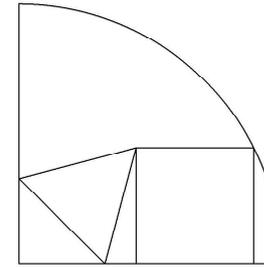
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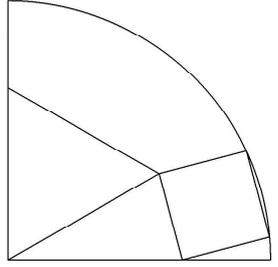
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(15) ▲



(16)



5. 答

1. (1) $2\sqrt{\frac{11-4\sqrt{3}}{73}}$ (2) $\frac{\sqrt{6}-\sqrt{2}}{2}$ (3) $\frac{\sqrt{6}-\sqrt{2}}{2}$ (4) $\frac{3-\sqrt{3}}{3}$

(5) $\frac{\sqrt{6}-\sqrt{2}}{2}$ (6) $2(2-\sqrt{3})$ (7) $\sqrt{\frac{5-2\sqrt{3}}{13}}$ (8) $\frac{\sqrt{6}-\sqrt{2}}{2}$

2. (1) $\frac{1}{2}$ (2) $\frac{\sqrt{5}}{5}$ (3) $\frac{\sqrt{6}-\sqrt{2}}{2}$ (4) $\frac{2(2\sqrt{2}-\sqrt{3})}{5}$

(5) $2\sqrt{\frac{13-6\sqrt{3}}{61}}$ (6) $-5+4\sqrt{2}-3\sqrt{3}+2\sqrt{6}$ (7) $\frac{2(2\sqrt{2}-\sqrt{3})}{5}$ (8) $\sqrt{\frac{4-\sqrt{3}}{13}}$

(9) $1249a^6 - (1334 + 756\sqrt{3})a^4 + (821 + 484\sqrt{3})a^2 - 4(41 + 12\sqrt{3}) = 0$

$$a = \sqrt{\frac{\sqrt[3]{c-d} + \sqrt[3]{c+d} + 1334 + 756\sqrt{3}}{3747}} \quad (\doteq 0.463938)$$

(ただし, $c = 364290551 - 774261702\sqrt{3}$, $d = 29976\sqrt{1894691580 - 744533199\sqrt{3}}$)

(10) $\sqrt{3}a^3 - a^2 - (2 + \sqrt{3})a + 2 = 0$

$$a = \frac{1}{3\sqrt{3}} \left[2\sqrt{10+6\sqrt{3}} \cos \left\{ \frac{2\pi}{3} - \frac{1}{3} \text{Arccos} \left(-\frac{\sqrt{-7142857+4123953\sqrt{3}}}{8\sqrt{2}} \right) \right\} + 1 \right]$$

($\doteq 0.529693$)

(11) $16 - 136a^2 + 381a^4 - 570a^6 + 1065a^8 - 1972a^{10} + 1901a^{12} - 814a^{14} + 121a^{16} = 0$

($a \doteq 0.474893$)

3. (1) $a = \frac{\sqrt{2}}{2}$, $b = \frac{2-\sqrt{2}}{2}$ (2) $a = \frac{\sqrt{2}}{2}$, $b = \frac{-\sqrt{6}+\sqrt{14}}{4}$

(3) $a = \frac{\sqrt{2}}{2}$, $b = \sqrt{\frac{5-\sqrt{2}-\sqrt{3+6\sqrt{2}}}{2}}$ (4) $a = \frac{\sqrt{10}}{5}$, $b = \frac{\sqrt{30}}{15}$

(5) $a = \frac{\sqrt{10}}{5}$, $b = \frac{\sqrt{30}-\sqrt{10}}{10}$ (6) $a = \frac{\sqrt{10}}{5}$, $b = \frac{\sqrt{15}-\sqrt{5}}{5}$

(7) $a = \frac{\sqrt{10}}{5}$, $b = \frac{-5+5\sqrt{3}+\sqrt{5}-\sqrt{15}}{5}$ (8) $a = \frac{\sqrt{10}}{5}$, $b = \frac{5\sqrt{3}-\sqrt{15}}{15}$

(9) $a = \frac{\sqrt{10}}{5}$, $b = \frac{3\sqrt{10}-\sqrt{30}}{10}$

(10) $a = \frac{\sqrt{10}}{5}$, $b = \sqrt{\frac{15-\sqrt{5}+\sqrt{15}-\sqrt{5(1-2\sqrt{3}+10\sqrt{5}+6\sqrt{15})}}{10}}$

(11) $a = \frac{\sqrt{10}}{5}$, $b = \frac{10\sqrt{3}-3\sqrt{30}}{15}$ (12) $a = \frac{\sqrt{10}}{5}$, $b = \frac{-3\sqrt{30}+2\sqrt{70}}{20}$

(13) $5a^4 - 2(1+3\sqrt{3})a^3 + (5+2\sqrt{3})a^2 + 2(-1+\sqrt{3})a - 2 = 0$ ($a \doteq 0.64251$),

$5b^8 - 42b^6 + (137-8\sqrt{3})b^4 + 8(-23+4\sqrt{3})b^2 + 16 = 0$ ($b \doteq 0.378309$)

(14) $a = \frac{6\sqrt{2}-\sqrt{6}}{11}$, $b = \frac{2(-1+2\sqrt{3})}{11}$

(15) $16 - 352a^2 + 3144a^4 - 14976a^6 + 42325a^8 - 74966a^{10} + 86132a^{12} - 66222a^{14} + 35437a^{16} - 13812a^{18} + 3944a^{20} - 736a^{22} + 64a^{24} = 0$ ($a \doteq 0.575106$), $-32 + 88b + 41b^2 - 294b^3 + 192b^4 + 142b^5 - 199b^6 + 40b^7 + 40b^8 - 24b^9 + 4b^{10} = 0$ ($b \doteq 0.556408$)

4. (1) $a = \frac{2\sqrt{2} - \sqrt{6}}{4}$, $b = 1$ (2) $a = \frac{-2\sqrt{3} + \sqrt{17}}{5}$, $b = 1$
- (3) $a = \sqrt{\frac{5-2\sqrt{3}}{13}}$, $b = 1$ (4) $a = \sqrt{\frac{5-2\sqrt{3}}{13}}$, $b = 1$
- (5) $a = \sqrt{\frac{5-2\sqrt{3}}{13}}$, $b = 1$ (6) $a = \frac{-2\sqrt{3} + \sqrt{17}}{5}$, $b = 1$
- (7) $a = \sqrt{\frac{13-6\sqrt{3}}{61}}$, $b = 1$ (8) $a = \sqrt{\frac{10+\sqrt{2}-2\sqrt{3}-3\sqrt{6}}{14}}$, $b = 1$
- (9) $a = \frac{\sqrt{6}-\sqrt{2}}{4}$, $b = \sqrt{3}-1$ (10) $a = \frac{\sqrt{6}-\sqrt{2}}{4}$, $b = \sqrt{3}-1$
- (11) $a = \frac{3\sqrt{2}-8\sqrt{6}+\sqrt{707+331\sqrt{3}}}{61}$, $b = \sqrt{3}-1$
- (12) $a = \frac{-40+5\sqrt{3}+\sqrt{3(965+436\sqrt{3})}}{122}$, $b = \sqrt{3}-1$
- (13) $-2(7+4\sqrt{3})+2(89+50\sqrt{3})a^2-5(97+52\sqrt{3})a^4+(529+248\sqrt{3})a^6-(251+74\sqrt{3})a^8+49a^{10}=0$ ($a \doteq 0.325942$), $b = \sqrt{3}-1$
- (14) $a = \sqrt{\frac{5-2\sqrt{3}}{13}}$, $b = 2\sqrt{\frac{5-2\sqrt{3}}{13}}$ (15) $a = \frac{\sqrt{5}}{5}$, $b = \frac{\sqrt{30}-\sqrt{10}}{5}$
- (16) $a = \sqrt{\frac{5-2\sqrt{3}}{13}}$, $b = \sqrt{\frac{4+\sqrt{3}}{13}}$

【お願い】他の配置があれば、ご連絡をお願いします。連絡先：tokioka@i4.gmob.jp

【参考文献】特になし

(2024/11/30)