四分円内の正方形と正三角形の１辺について　Ⅱ

数実研会員　時岡郁夫

半径１の四分円内に正方形甲２個，正方形甲乙，正三角形甲２個，正三角形甲乙をいろいろな配置で内接させる。甲，乙の１辺の長さをそれぞれ a, b とおいて求めてみた。

１．正方形甲２個　　８通り

２．正方形甲，乙　　15 通り

３．正三角形甲２個　14 通り

４．正三角形甲，乙　14 通り

ただし，

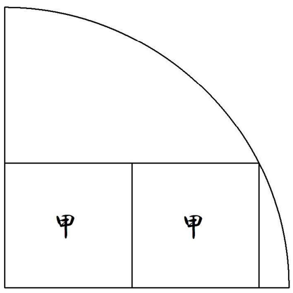
■印の問題の正方形甲と▲印の問題の正三角形甲は，四分円の中心角の二等分線に関して対称である。

★印の問題は，a または b の値が複雑になるか或いは代数的に求めることができないことが分かった。

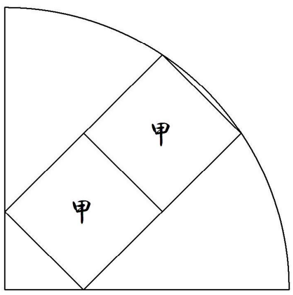
また，２(13) は，四分円の中心と２個の甲の共有点との距離が d の場合である。

１．正方形甲２個 ８通り

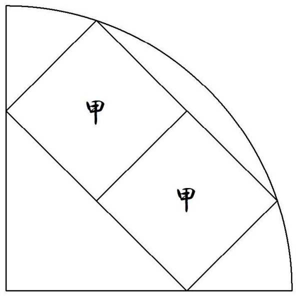
(1)



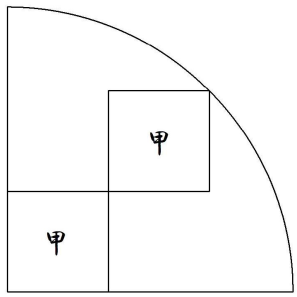
(2)



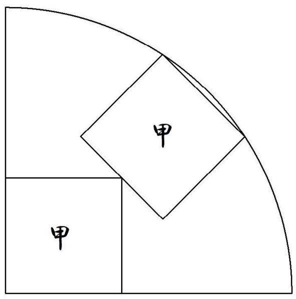
(3)



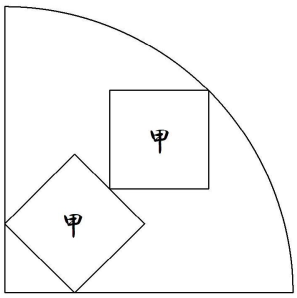
(4) ■



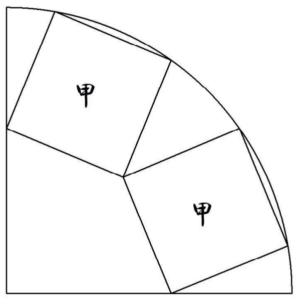
(5) ■



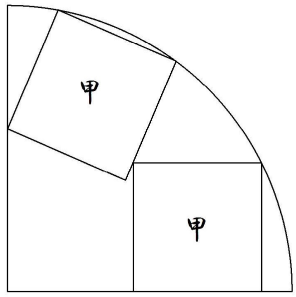
(6) ■



(7)

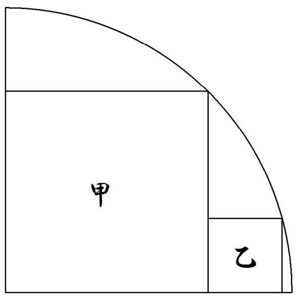


(8) ★

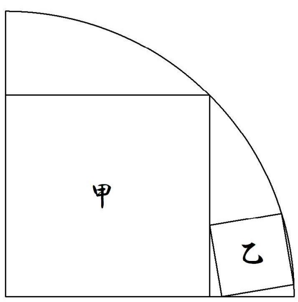


２．正方形甲，乙　15 通り

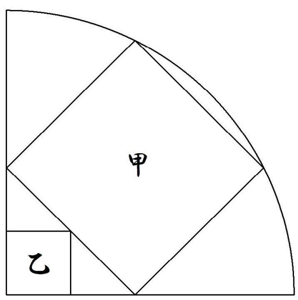
(1)



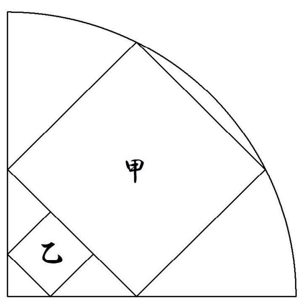
(2) ★



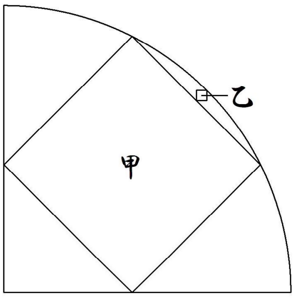
(3)



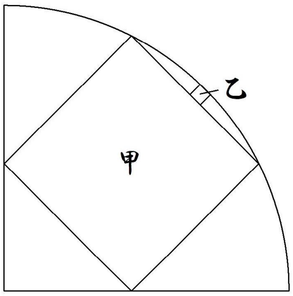
(4)



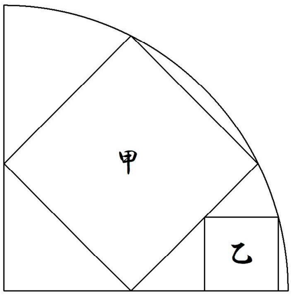
(5) ■



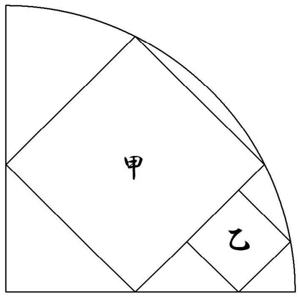
(6) ■



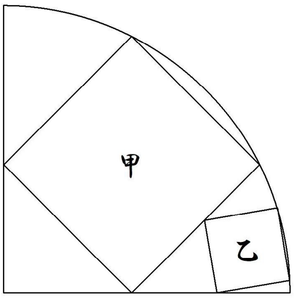
(7)



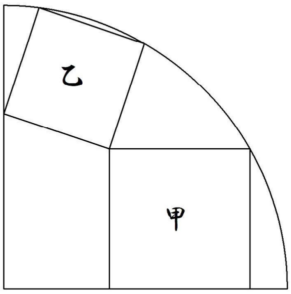
(8)



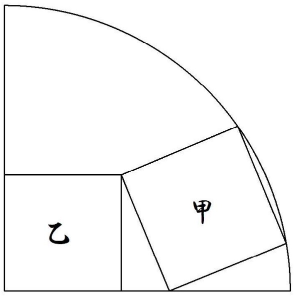
(9)



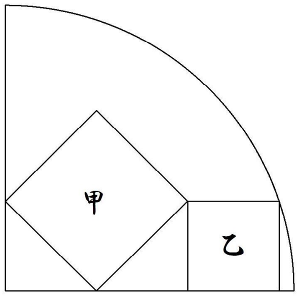
(10)



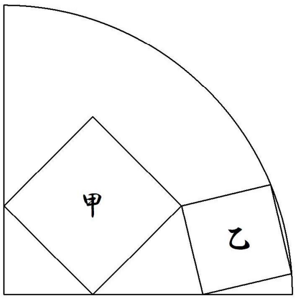
(11)



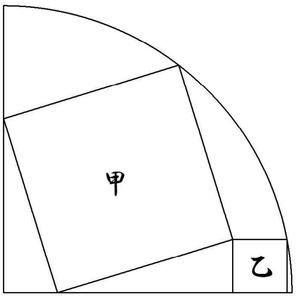
(12) ■



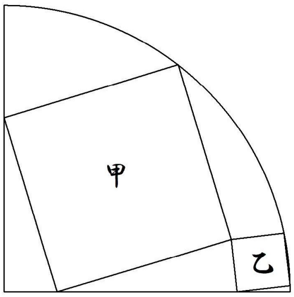
(13) ■



(14)

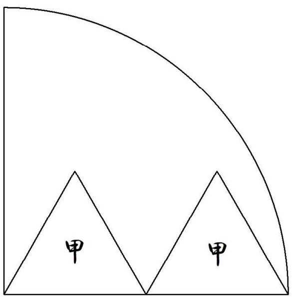


(15) ★

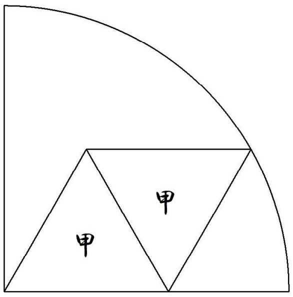


３．正三角形甲２個　14 通り

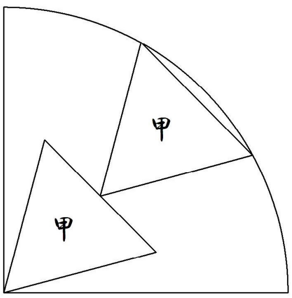
(1)



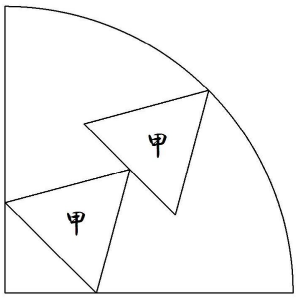
(2)



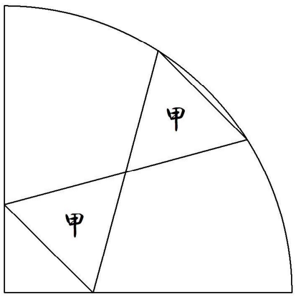
(3) ▲



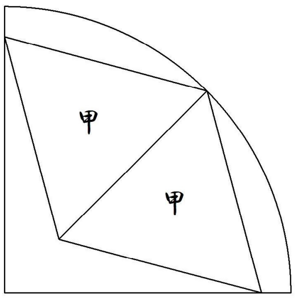
(4) ▲



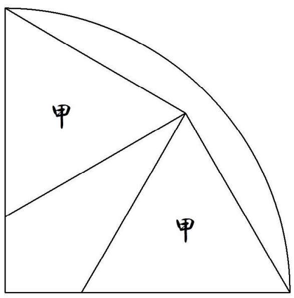
(5) ▲



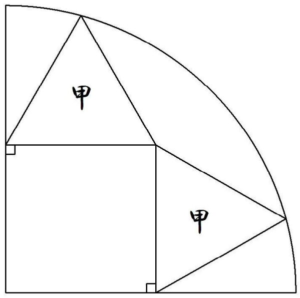
(6) ▲



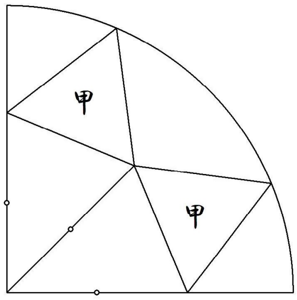
(7)



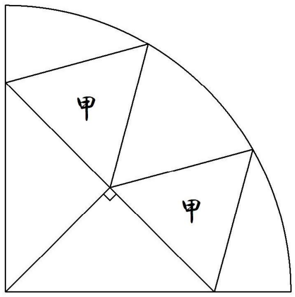
(8)



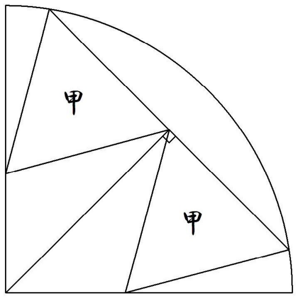
(9)



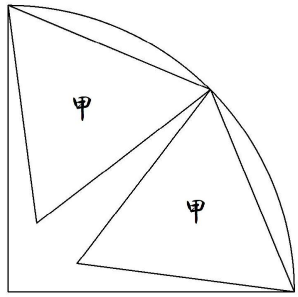
(10)



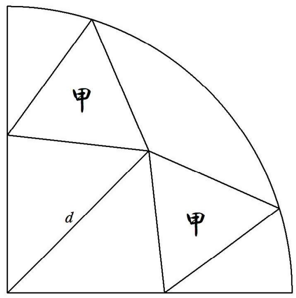
(11)



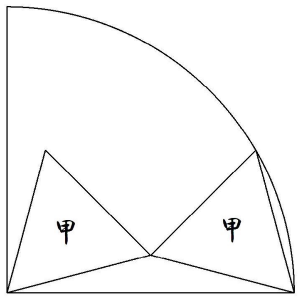
(12)



(13)

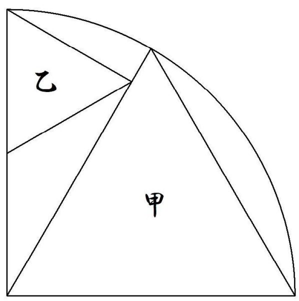


(14)

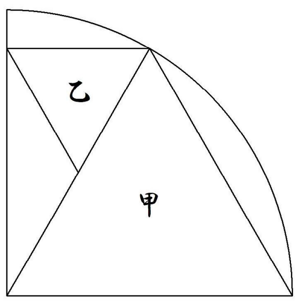


4．正三角形甲，乙　14 通り

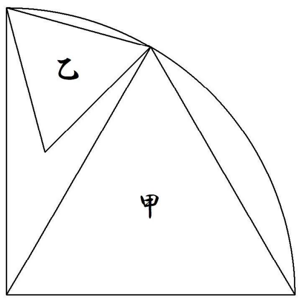
(1)



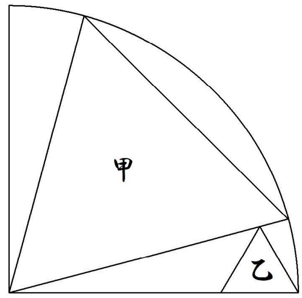
(2)



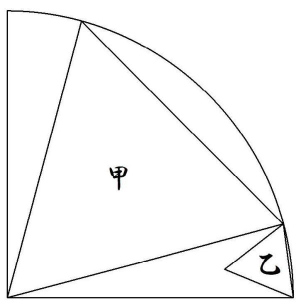
(3)



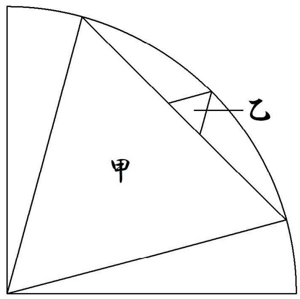
(4)　▲



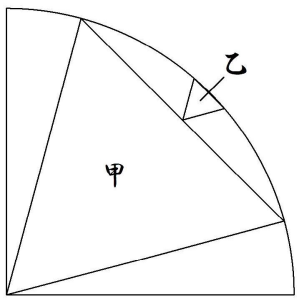
(5)　▲



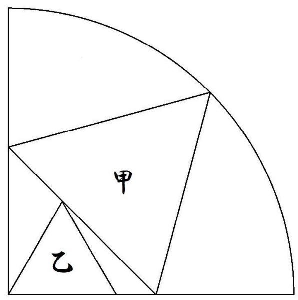
(6)　▲



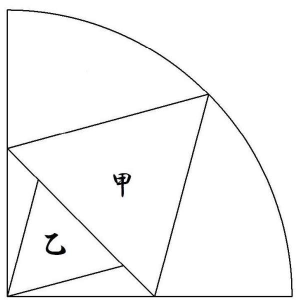
(7)　▲



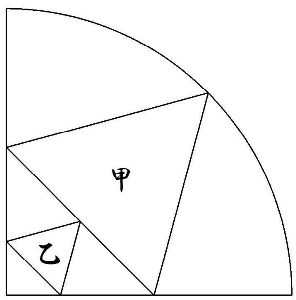
(8)　▲



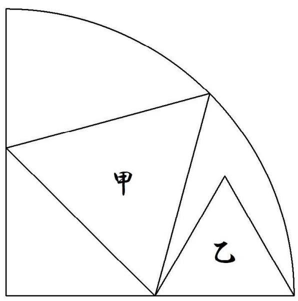
(9)　▲



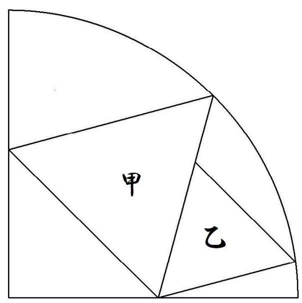
(10)　▲



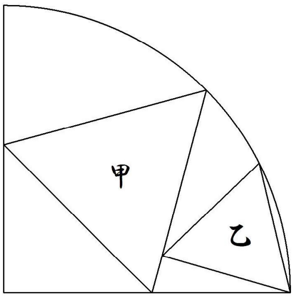
(11)　▲



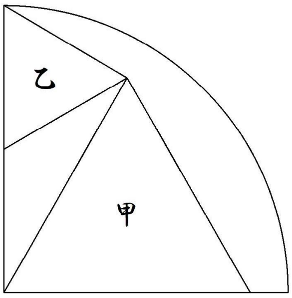
(12)　▲



(13) ▲



(14)



５．答

１．(1) $\dfrac{\sqrt{5}}{5}$　(2) $\dfrac{\sqrt{26}}{13}$　(3) $\dfrac{\sqrt{5}}{5}$　(4) $\dfrac{\sqrt{2}}{4}$　(5) $2\sqrt{\dfrac{13-8\sqrt{2}}{41}}$　(6) $2(3-2\sqrt{2})$

(7) $\sqrt{\dfrac{2-\sqrt{2}}{3}}$

(8) 甲の１辺 a を解にもつ高次方程式の１つは,

$$256-3584a^2+19840a^4-57088a^6+94384a^8-93760a^{10}+56152a^{12}-19128a^{14}+2929a^{16}=0$$

（ $a \fallingdotseq 0.485352$ ）

２．(1) 甲：$\dfrac{\sqrt{2}}{2}$，乙：$\dfrac{-\sqrt{2}+\sqrt{6}}{4}$

(2) 甲：$\dfrac{\sqrt{2}}{2}$，乙の１辺 b を解にもつ高次方程式の１つは，$25b^8-124b^6+156b^4-72b^2+4=0$

題意に適する解は，$b=\dfrac{1}{5}\sqrt{31+\sqrt{311+m}-\sqrt{622-m+\dfrac{10382}{\sqrt{311+m}}}}$　（$\fallingdotseq 0.252941$）

（ただし，$m=\dfrac{25}{3}\left(\sqrt[3]{-7074+762\sqrt{87}}-\sqrt[3]{7074+762\sqrt{87}}\right)$）

(3) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{\sqrt{5}}{10}$　(4) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{\sqrt{10}}{15}$　(5) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{5\sqrt{2}-3\sqrt{5}}{10}$

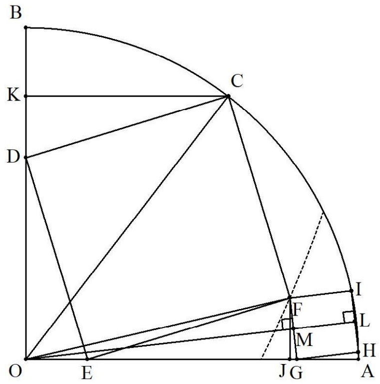
(6) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{\sqrt{410}-6\sqrt{10}}{25}$　(7) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{2(\sqrt{30}-\sqrt{5})}{25}$

(8) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{2\sqrt{10}}{25}$　(9) 甲：$\dfrac{\sqrt{10}}{5}$，乙：$\dfrac{\sqrt{7-\sqrt{41}}}{3}$　(10) 甲：$\dfrac{4\sqrt{65}}{65}$，乙：$\dfrac{\sqrt{26}}{13}$

(11) 甲：$\dfrac{\sqrt{6-3\sqrt{2}}}{3}$，乙：$\dfrac{\sqrt{6}}{6}$　(12) 甲：$\dfrac{\sqrt{5}}{5}$，乙：$\dfrac{\sqrt{10}}{10}$

(13) 甲：$\sqrt{\dfrac{2(15+2\sqrt{5})}{205}}$，乙：$\sqrt{\dfrac{2(11-4\sqrt{5})}{41}}$　(14) 甲：$\sqrt{\dfrac{15-4\sqrt{5}}{15}}$，乙：$\sqrt{\dfrac{5-2\sqrt{5}}{15}}$

(15)　甲：$\sqrt{\dfrac{1+2d^2-d\sqrt{2-d^2}}{2}}$　（$\fallingdotseq 0.636036$）　ただし，図で，KD＝OE＝GF＝d とおくと，d は，



$$
\begin{aligned}
&(1-48b^2+72b^4-64b^6+16b^8)^2\\
&-8(-5+296b^2+60b^4-2240b^6+4048b^8-2432b^{10}+576b^{12})d^2\\
&+4(219-1800b^2+4484b^4-1984b^6+528b^8+384b^{10}+704b^{12})d^4\\
&-8(-1199+11136b^2-21448b^4+16704b^6-2608b^8+1280b^{10})d^6\\
&+2(29067-84560b^2+74008b^4+11584b^6+2224b^8)d^8\\
&+8(2267-2504b^2-10220b^4+896b^6)d^{10}\\
&+4(3299+5192b^2+588b^4)d^{12}-200(-9+16b^2)d^{14}+625d^{16}=0
\end{aligned}
$$

の解（$d \fallingdotseq 0.184978$）

$\left(\text{ただし，} b=\sqrt{\dfrac{2(105-6\sqrt[3]{2}-56\sqrt[3]{4})}{493}}\right)$，乙：$\sqrt{\dfrac{2(105-6\sqrt[3]{2}-56\sqrt[3]{4})}{493}}$

3．(1)　$\dfrac{1}{2}$　(2)　$\dfrac{\sqrt{3}}{3}$　(3)　$\dfrac{2\sqrt{13}}{13}$　(4)　$\dfrac{2(2\sqrt{3}-1)}{11}$　(5)　$\sqrt{\dfrac{2(7-2\sqrt{3})}{37}}$　(6)　$\dfrac{\sqrt{3}}{3}$

(7)　$\sqrt{3}-1$　(8)　$\dfrac{\sqrt{6}-\sqrt{2}}{2}$　(9)　$\dfrac{2+\sqrt{2}-\sqrt{6}}{2}$　(10)　$\dfrac{\sqrt{6}-\sqrt{2}}{2}$　(11)　$\sqrt{\dfrac{2(4-\sqrt{3})}{13}}$

(12)　$\sqrt{2-\sqrt{2}}$　(13)　$\sqrt{\dfrac{2+d^2-d\sqrt{8-4\sqrt{3}-(7+4\sqrt{3})d^2}}{2}}$　(14)　$\dfrac{\sqrt{6}-\sqrt{2}}{2}$

4．(1)　甲：1，乙：$\dfrac{1}{2}$　(2)　甲：1，乙：$\dfrac{1}{2}$　(3)　甲：1，乙：$\dfrac{\sqrt{6}-\sqrt{2}}{2}$

(4)　甲：1，乙：$2-\sqrt{3}$　(5)　甲：1，乙：$\dfrac{\sqrt{2(4-\sqrt{2}-\sqrt{6})}}{2}$　(6)　甲：1，乙：$\dfrac{2\sqrt{3}-3}{3}$

(7)　甲：1，乙：$\dfrac{-3+\sqrt{13}}{4}$　(8)　甲：$\sqrt{3}-1$，乙：$2\sqrt{2}-\sqrt{6}$

(9)　甲：$\sqrt{3}-1$，乙：$\dfrac{3-\sqrt{3}}{3}$　(10)　甲：$\sqrt{3}-1$，乙：$2\sqrt{3}-3$

(11)　甲：$\sqrt{3}-1$，乙：$\dfrac{2+\sqrt{2}-\sqrt{6}}{2}$　(12)　甲：$\sqrt{3}-1$，乙：$\dfrac{-1+\sqrt{4\sqrt{3}-3}}{2}$

(13)　甲：$\sqrt{3}-1$，乙：$b=\dfrac{\sqrt{6+\sqrt{2}+2\sqrt{3}-\sqrt{6}-2\sqrt{2+4\sqrt{2}+\sqrt{3}+2\sqrt{6}}}}{2}$

(14)　甲：$\dfrac{\sqrt{3}}{2}$，乙：$b=\dfrac{1}{2}$

【お願い】他の配置があれば，ご連絡をお願いします。連絡先： tokioka@i4.gmobb.jp

【参考文献】特になし

(2024/11/30)