

四分円内の 4 円の半径について

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半径 1 の四分円内に、図のように 4 個の円（甲、乙、丙等）を配置し、それぞれの円の半径を求めてみた。

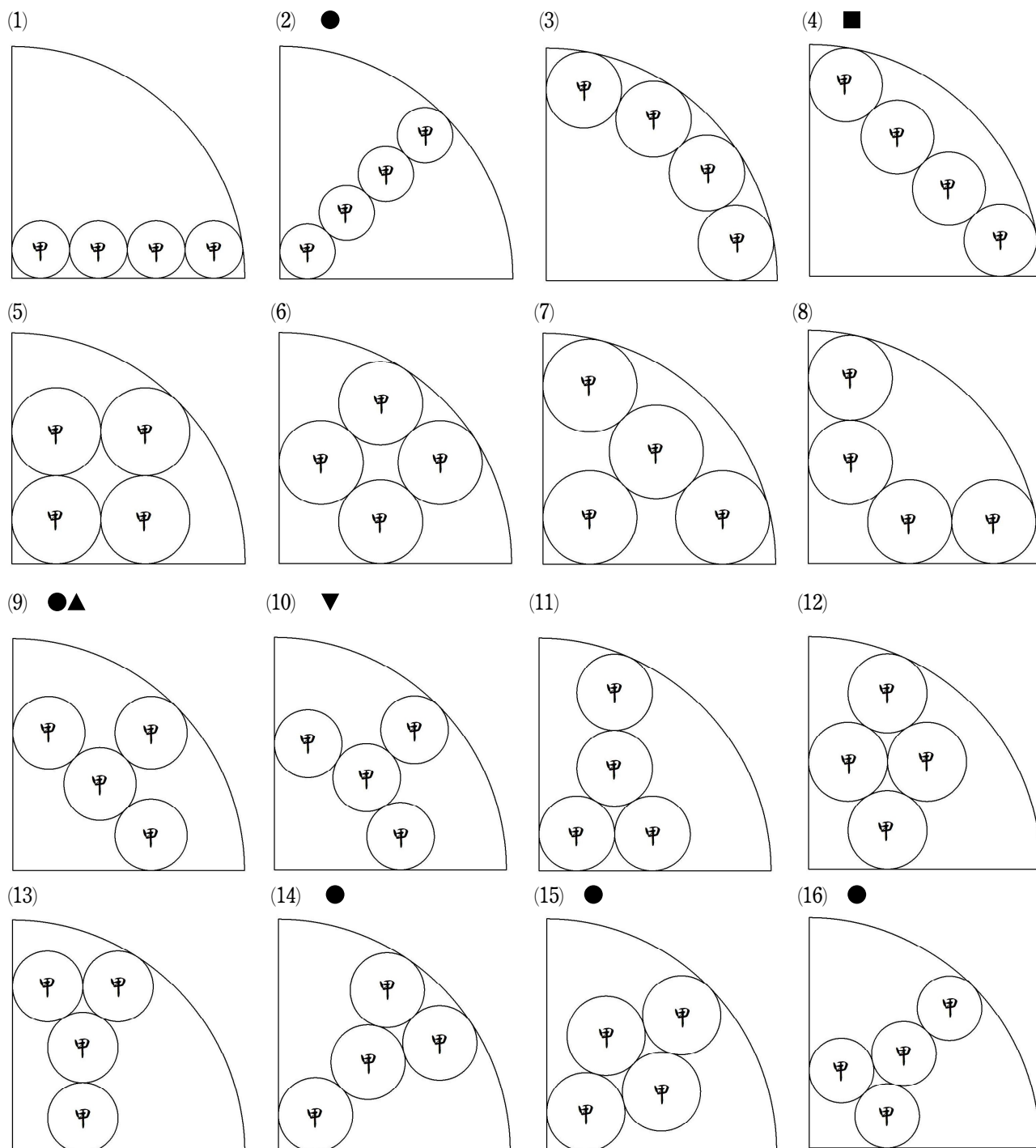
ただし、●印：図形は四分円の中心角の二等分線に関して対称，■印：4 個の円の中心は同一直線上，

◆印：3 個の甲円は四分円の中心角の二等分線に関して対称，▲印：3 個の円の中心は同一直線上，

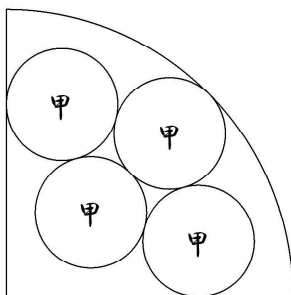
▼印：3 個の円の中心は四分円の中心から等距離，○印：3 個の円の中心は四分円の中心角の二等分線上

とする。

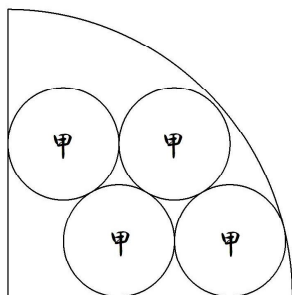
なお、(53) は甲円の半径の満たす方程式が 5 次となり、求めることができなかった。



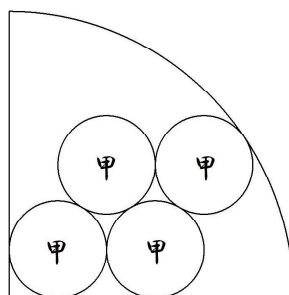
(17) ●



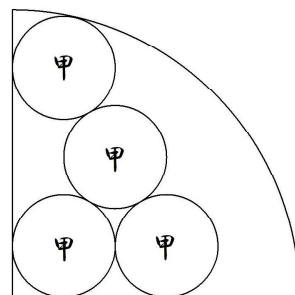
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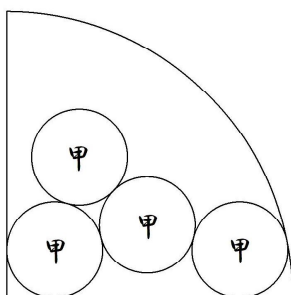
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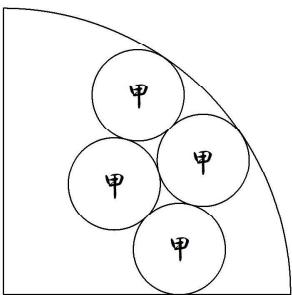
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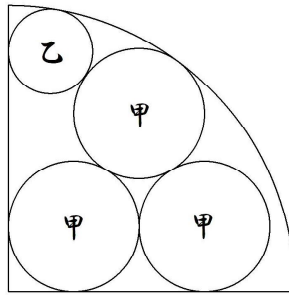
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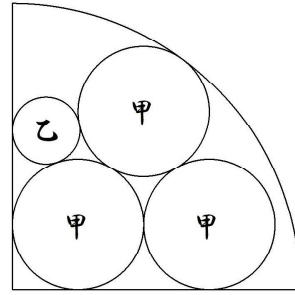
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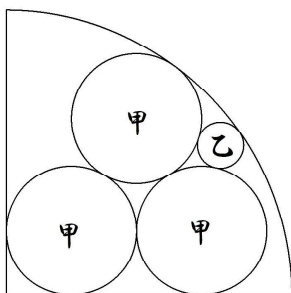
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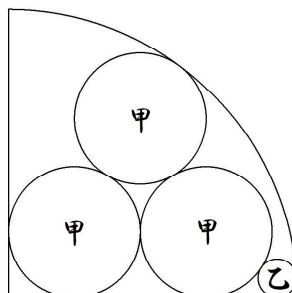
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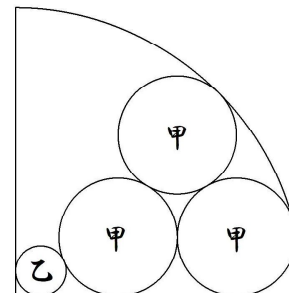
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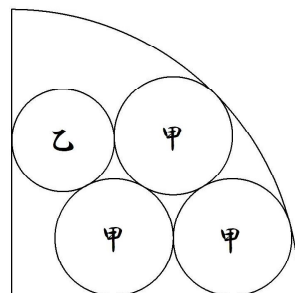
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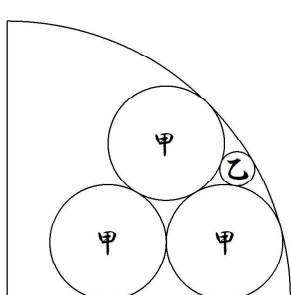
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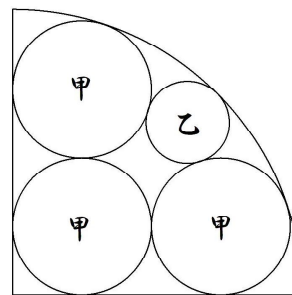
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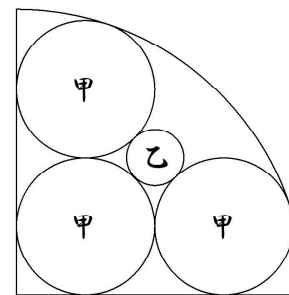
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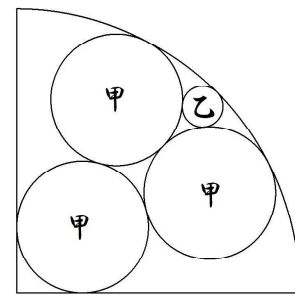
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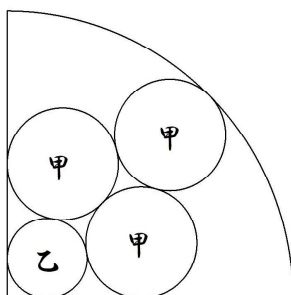
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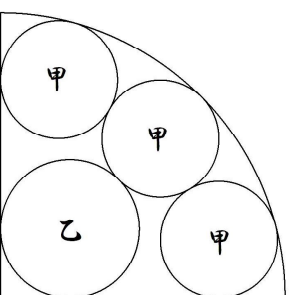
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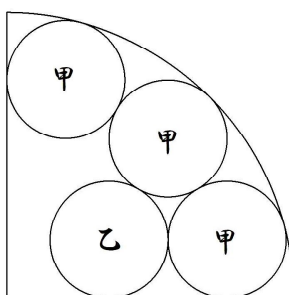
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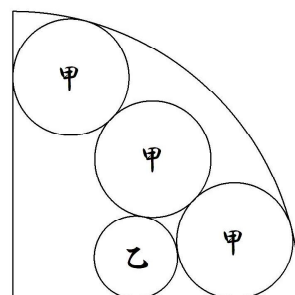
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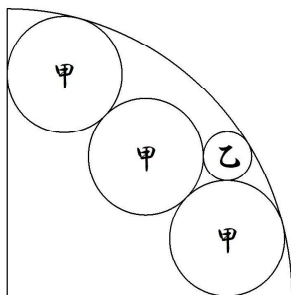
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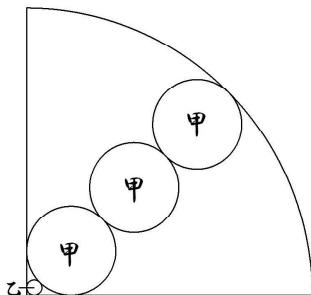
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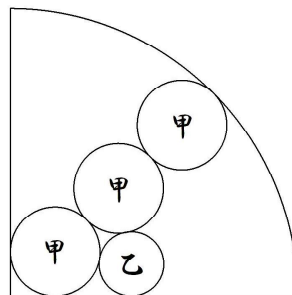
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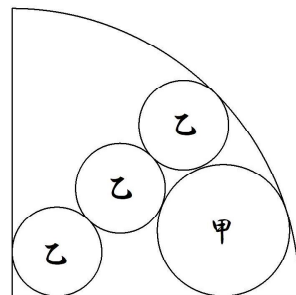
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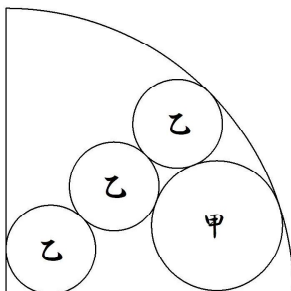
(39) ○



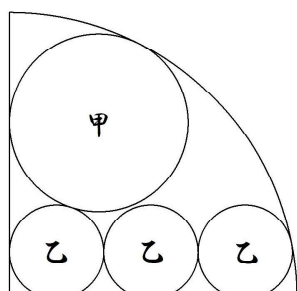
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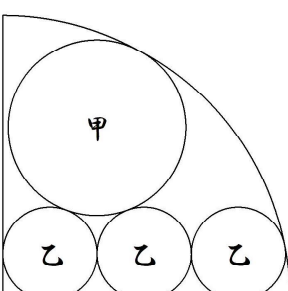
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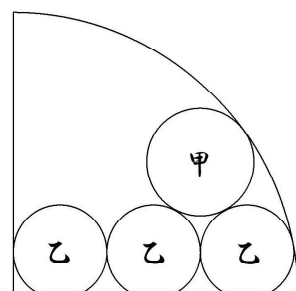
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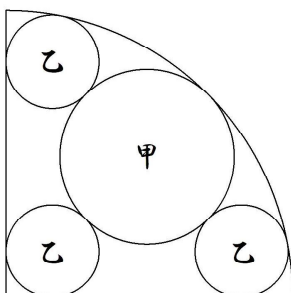
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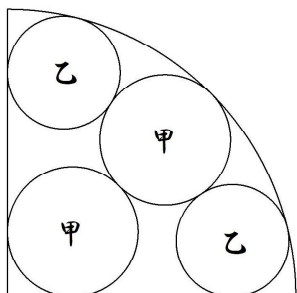
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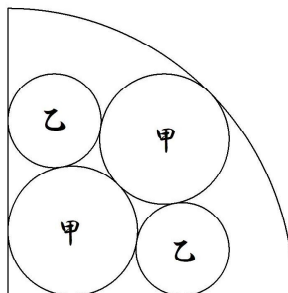
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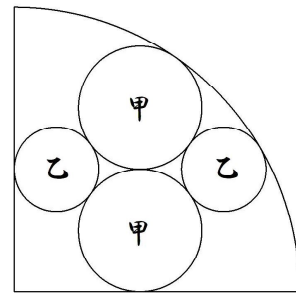
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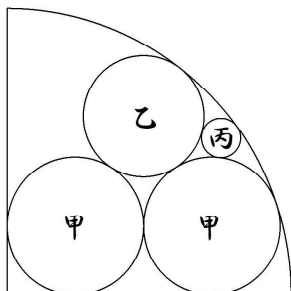
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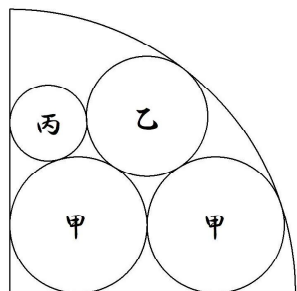
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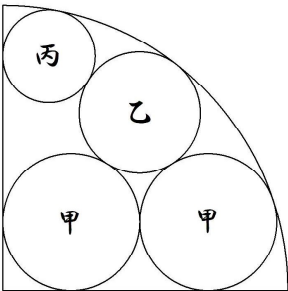
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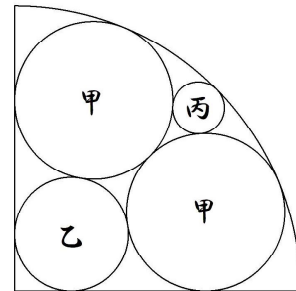
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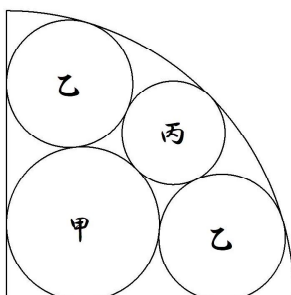
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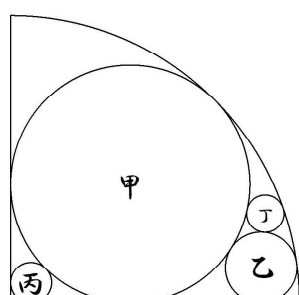
(52)



(53)



(54)



答

- (1) $\frac{-1+5\sqrt{2}}{49}$ ($\doteq 0.123899$) (2) $\frac{7-\sqrt{2}}{47}$ ($\doteq 0.118847$)
- (3) $-7-4\sqrt{2}+2\sqrt{20+14\sqrt{2}}+\sqrt{168+116\sqrt{2}-2\sqrt{13780+9742\sqrt{2}}}$ ($\doteq 0.163243$)
- (4) $\frac{-19+6\sqrt{2}+\sqrt{2(2962-981\sqrt{3})}}{289}$ ($\doteq 0.157799$)
- (5) $\frac{3\sqrt{2}-1}{17}$ ($\doteq 0.190744$) (6) $\frac{-11+6\sqrt{2}+\sqrt{6(122-71\sqrt{2})}}{49}$ ($\doteq 0.180961$)
- (7) $\frac{-9+4\sqrt{2}+\sqrt{2(277-134\sqrt{2})}}{49}$ ($\doteq 0.20174$) (8) $\frac{-11+6\sqrt{2}+\sqrt{6(122-61\sqrt{2})}}{49}$ ($\doteq 0.180961$)
- (9) $\frac{5-\sqrt{2}}{23}$ ($\doteq 0.155904$) (10) $\frac{-3+13\sqrt{2}-12\sqrt{3}+5\sqrt{6}}{47}$ ($\doteq 0.145694$)
- (11) $\frac{-5+2\sqrt{3}+\sqrt{2(116-49\sqrt{3})}}{39}$ ($\doteq 0.162937$) (12) $\frac{-4+\sqrt{3}+\sqrt{227-60\sqrt{3}}}{52}$ ($\doteq 0.169732$)
- (13) $\frac{-10+3\sqrt{3}+\sqrt{3(529-166\sqrt{3})}}{146}$ ($\doteq 0.151449$)
- (14) $\frac{-177+52\sqrt{2}+68\sqrt{3}-14\sqrt{6}+\sqrt{2(72709-25814\sqrt{2}-31478\sqrt{3}+9717\sqrt{6})}}{529}$ ($\doteq 0.160192$)
- (15) $\frac{-13-11\sqrt{2}+18\sqrt{3}+4\sqrt{6}}{73}$ ($\doteq 0.170117$) (16) $\frac{44-17\sqrt{2}-15\sqrt{3}+8\sqrt{6}}{97}$ ($\doteq 0.139933$)
- (17) $\frac{2+5\sqrt{2}+3\sqrt{3}-4\sqrt{6}}{23}$ ($\doteq 0.194316$) (18) $\frac{-1+\sqrt{17}}{16}$ ($\doteq 0.195194$)
- (19) $\frac{-19+2\sqrt{3}+\sqrt{2(3502-387\sqrt{3})}}{349}$ ($\doteq 0.171116$) (20) $\frac{-13+4\sqrt{3}+\sqrt{2(895-294\sqrt{3})}}{121}$ ($\doteq 0.179381$)
- (21) $-201-142\sqrt{2}+116\sqrt{3}+82\sqrt{6}+\sqrt{2(80821+57149\sqrt{2}-46662\sqrt{3}-32995\sqrt{6})}$ ($\doteq 0.167632$)
- (22) $\frac{-177+52\sqrt{2}+68\sqrt{3}-14\sqrt{6}+\sqrt{2(72709-25814\sqrt{2}-31478\sqrt{3}+9717\sqrt{6})}}{529}$ ($\doteq 0.160192$)
- (23) 甲: $\frac{-7+2\sqrt{3}+\sqrt{2(160-51\sqrt{3})}}{37}$ ($\doteq 0.228005$), 乙: $\frac{5+11\sqrt{3}+2\sqrt{2(640-113\sqrt{3})}}{169}$ ($\doteq 0.495088$)
- (24) 甲: $\frac{-7+2\sqrt{3}+\sqrt{2(160-51\sqrt{3})}}{37}$ ($\doteq 0.228005$),
乙: $\frac{3\{-21+14\sqrt{2}+6\sqrt{3}-4\sqrt{6}+\sqrt{2(2720-1920\sqrt{2}-867\sqrt{3}+612\sqrt{6})}\}}{37}$ ($\doteq 0.117358$)
- (25) 甲: $\frac{-7+2\sqrt{3}+\sqrt{2(160-51\sqrt{3})}}{37}$ ($\doteq 0.228005$),
乙: $\frac{2862415-112190\sqrt{3}-7\sqrt{2(15384363172-6433792093\sqrt{3})}-2\sqrt{d}}{4894693}$ ($\doteq 0.0811064$)
ただし, $d = 37\{111156901555-33248781091\sqrt{3}-26\sqrt{39(501892638334251377-261866051982917458\sqrt{3})}\}$
- (26) 甲: $\frac{-7+2\sqrt{3}+\sqrt{2(160-51\sqrt{3})}}{37}$ ($\doteq 0.228005$),
乙: $\frac{-77284-22821\sqrt{3}+\sqrt{7725935456+3589331146\sqrt{3}}-d}{4477}$ ($\doteq 0.0667881$)
ただし, $d = 6\sqrt{37(11387007+5369319\sqrt{3}-2\sqrt{54034061448737+30568394410458\sqrt{3}})}$

- (27) 甲： $-7 - 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$ ($\doteq 0.205605$) ,
 乙： $-2 - \sqrt{2} + \sqrt{3} + \sqrt{6} - 4\sqrt{-7 - 12\sqrt{2} + 23\sqrt{3} + 8\sqrt{6}}$ ($\doteq 0.08876426$)
- (28) 甲： $-7 - 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$ ($\doteq 0.205605$) ,
 乙： $13 + 11\sqrt{2} - 8\sqrt{3} - 5\sqrt{6} - 2\sqrt{-117 - 84\sqrt{2} + 69\sqrt{3} + 48\sqrt{6}}$ ($\doteq 0.178226$)
- (29) 甲： $-7 - 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$ ($\doteq 0.205605$) ,
 乙： $\frac{67 - 534\sqrt{2} + 480\sqrt{3} + 234\sqrt{6} - 2\sqrt{182463 + 1160\sqrt{2} + 25444\sqrt{3} - 54840\sqrt{6}}}{1679}$ ($\doteq 0.0617608$)
- (30) 甲： $\frac{-1 + \sqrt{10}}{9}$ ($\doteq 0.240253$) , 乙： $\frac{17 + 6\sqrt{2} - 8\sqrt{5} - \sqrt{10}}{31}$ ($\doteq 0.143047$)
- (31) 甲： $\frac{-1 + \sqrt{10}}{9}$ ($\doteq 0.240253$) , 乙： $\frac{1 - \sqrt{2} + 2\sqrt{5} - \sqrt{10}}{9}$ ($\doteq 0.0995161$)
- (32) 甲： $-5 + 2\sqrt{6} + \sqrt{54 - 22\sqrt{6}}$ ($\doteq 0.232485$) ,
 乙： $\frac{19 + 250\sqrt{2} - 70\sqrt{3} + 240\sqrt{6} - 2\sqrt{180150 - 50400\sqrt{2} + 67056\sqrt{3} - 32942\sqrt{6}}}{1103}$ ($\doteq 0.071967$)
- (33) 甲： $\frac{2 + 5\sqrt{2} + 3\sqrt{3} - 4\sqrt{6}}{23}$ ($\doteq 0.194316$) ,
 乙： $\frac{16 + 17\sqrt{2} + \sqrt{3} - 9\sqrt{6} - 2\sqrt{250 + 73\sqrt{2} - 108\sqrt{3} - 40\sqrt{6}}}{23}$ ($\doteq 0.139654$)
- (34) 甲： $-7 - 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$ ($\doteq 0.205605$) , 乙： $5 + 5\sqrt{2} - 4\sqrt{3} - 2\sqrt{6}$ ($\doteq 0.243885$)
- (35) 甲： $-7 - 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$ ($\doteq 0.205605$) , 乙： 甲と同じ
- (36) 甲： $\frac{-7 + 2\sqrt{3} + \sqrt{2(160 - 51\sqrt{3})}}{37}$ ($\doteq 0.228005$) , 乙： $\frac{5 + 11\sqrt{3} + 2\sqrt{2(640 - 113\sqrt{3})}}{169}$ ($\doteq 0.495088$)
- (37) 甲： $\frac{-9 + 4\sqrt{2} + \sqrt{2(277 - 134\sqrt{2})}}{49}$ ($\doteq 0.20174$) ,
 乙： $\frac{58 - 4\sqrt{2} - \sqrt{2(277 - 134\sqrt{2})} - 7\sqrt{76 - 12\sqrt{2} - 3\sqrt{2(277 - 134\sqrt{2})}}}{98}$ ($\doteq 0.0849724$)
- (38) 甲： $\frac{5 - \sqrt{2}}{23}$ ($\doteq 0.155904$) , 乙： $\frac{19 - 13\sqrt{2}}{23}$ ($\doteq 0.0267489$)
- (39) 甲： $\frac{5 - \sqrt{2}}{23}$ ($\doteq 0.155904$) , 乙： $\frac{13 + 2\sqrt{2} - 2\sqrt{34 + 7\sqrt{2}}}{23}$ ($\doteq 0.112047$)
- (40) 甲： $\frac{5 - \sqrt{2}}{23}$ ($\doteq 0.155904$) , 乙： $\frac{14469 - 649\sqrt{2} - 4\sqrt{762151 + 412436\sqrt{2}}}{38663}$ ($\doteq 0.230491$)
- (41) 甲： $\frac{5 - \sqrt{2}}{23}$ ($\doteq 0.155904$) , 乙： $\frac{2(53 - 6\sqrt{2})}{391}$ ($\doteq 0.227697$)
- (42) 甲： $\frac{-1 + \sqrt{26}}{25}$ ($\doteq 0.163961$) , 乙： $\frac{4259 - 309\sqrt{26} - 10\sqrt{-48922 + 9972\sqrt{26}}}{7225}$ ($\doteq 0.310672$)
- (43) 甲： $\frac{-1 + \sqrt{26}}{25}$ ($\doteq 0.163961$) , 乙： $\frac{4637 - 112\sqrt{26} - 2\sqrt{67447 + 16578\sqrt{26}}}{10625}$ ($\doteq 0.309291$)
- (44) 甲： $\frac{-1 + \sqrt{26}}{25}$ ($\doteq 0.163961$) , 乙： $\frac{217 - 12\sqrt{2} - 26\sqrt{13} + 58\sqrt{26}}{2125}$ ($\doteq 0.18919$)
- (45) 甲： $\frac{4 + 3\sqrt{2} - 2\sqrt{5} - \sqrt{10}}{2}$ ($\doteq 0.304114$) , 乙： $-3 + \sqrt{10}$ ($\doteq 0.166228$)

$$(46) \text{ 甲: } \frac{3-\sqrt{2}}{7} \quad (\doteq 0.226541), \text{ 乙: } \frac{83+11\sqrt{2}-2\sqrt{410+23\sqrt{2}}}{289} \quad (\doteq 0.195445)$$

$$(47) \text{ 甲: } \frac{3-\sqrt{2}}{7} \quad (\doteq 0.226541), \text{ 乙: } \frac{7-2\sqrt{10-\sqrt{2}}}{7} \quad (\doteq 0.162814)$$

$$(48) \text{ 甲: } \frac{1}{363} \left[-1 - \sqrt{28681} \cos \left\{ \frac{2\pi}{3} - \frac{1}{3} \operatorname{Arccos} \left(-\frac{4829723}{28681\sqrt{28681}} \right) \right\} \right] \quad (\doteq 0.216024),$$

$$\text{乙: } \frac{1}{242} \left[-123 + 2\sqrt{28681} \cos \left\{ \frac{2\pi}{3} - \frac{1}{3} \operatorname{Arccos} \left(-\frac{4829723}{28681\sqrt{28681}} \right) \right\} \right] \quad (\doteq 0.148072)$$

$$(49) \text{ 甲: } \frac{-1+\sqrt{10}}{9} \quad (\doteq 0.240253), \text{ 乙: } \frac{-17+8\sqrt{10}-2\sqrt{223-70\sqrt{10}}}{27} \quad (\doteq 0.212464),$$

$$\text{丙: } \frac{1011-708\sqrt{2}+414\sqrt{5}-289\sqrt{10}}{311} \quad (\doteq 0.0693592)$$

$$(50) \text{ 甲: } \frac{-1+\sqrt{10}}{9} \quad (\doteq 0.240253), \text{ 乙: } \frac{-17+8\sqrt{10}-2\sqrt{223-70\sqrt{10}}}{27} \quad (\doteq 0.212464),$$

$$\text{丙: } \frac{-33-22\sqrt{2}+8\sqrt{5}+15\sqrt{10}}{9} \quad (\doteq 0.134446)$$

$$(51) \text{ 甲: } \frac{-1+\sqrt{10}}{9} \quad (\doteq 0.240253), \text{ 乙: } \frac{-17+8\sqrt{10}-2\sqrt{223-70\sqrt{10}}}{27} \quad (\doteq 0.212464),$$

$$\text{丙: } -3+\sqrt{10} \quad (\doteq 0.162278)$$

$$(52) \text{ 甲: } -3+2\sqrt{2}+\sqrt{2(10-7\sqrt{2})} \quad (\doteq 0.276769), \text{ 乙: } -1-\sqrt{2}+\sqrt{2(2+\sqrt{2})} \quad (\doteq 0.198912),$$

$$\text{丙: } \frac{13+10\sqrt{2}-2\sqrt{2(50+17\sqrt{2})}}{31} \quad (\doteq 0.0904594)$$

$$(53) \text{ 甲: } r_1, \text{ 乙: } r_2, \text{ 丙: } r_3 \text{ とおく。}$$

$$7r_1^5 + (241+181\sqrt{2})r_1^4 - 2(607+430\sqrt{2})r_1^3 - 2(201+143\sqrt{2})r_1^2 + 3(97+68\sqrt{2})r_1 - 27 - 19\sqrt{2} = 0$$

$$\text{近似値は, } r_1 \doteq 0.265916, \quad r_2 \doteq 0.219455, \quad r_3 \doteq 0.179011$$

$$(54) \text{ 甲: } -1+\sqrt{2} \quad (\doteq 0.414214), \text{ 乙: } \frac{-1+5\sqrt{2}}{49} \quad (\doteq 0.123899), \text{ 丙: } -7+5\sqrt{2} \quad (\doteq 0.0710678),$$

$$\text{丁: } \frac{-7+6\sqrt{2}}{23} \quad (\doteq 0.0645775)$$

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