Let's Try Complex Change

Complex change to a figure using Decimal BASIC

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The report ," The Rut of Mobius " and " The Tail of Mobius ", which was written by Mr.Nakamura teacher from Sinkawa Senior High school, explains the power change in the complex number plane.

In this report, an attempts is made to draw a figure and show how a complex change modifies the figure on the screen using Decimal BASIC.

Just trace a simple figure on the screen with a mouse pen, and the program will display the modified figure simultaneously.

Let's try to enjoy such complex changes.

1. Let's try to picture using Decimal BASIC

Consider a square with sides of 1 unit length.

Let's try to picture how the diagram of this figure is changed using complex function

$w=z^2$.

Choose

OPTION ARITHMETIC COMPLEX

in the introduction of the program in Decimal BASIC in order to set to complex number mode.

Then a complex number,

$z=x+iy$

can be written are gotten in $z=$complex(x,y).

And, by defining a function as

$w=z^2$,

a complex number after the change can be found.

Then describe the real and imaginary part,

$\mathrm{Re}(w), \mathrm{Im}(w)$

After taking out the real part and the imaginary part, let's picture the figure after the change.

Consider how the tracing of a squre figure is modified by this program.

```
        OPTION  ARITHMETIC  COMPLEX                    ! It sets to the complex number mode

        SET WINDOW -2,2,-1,3

        DRAW grid

        SET LINE STYLE 1

        SET LINE COLOR 1

        PLOT LINES: 0,0 ; 1,0 ; 1,1 ; 0,1; 0,0         !The original diagram

        SET POINT STYLE 3

        Do

           MOUSE POLL  x,y,left,right

            if right=1  then exit  do                  !It ends by the mouse right button click.

           if left=1  then

              SET POINT COLOR 1

              PLOT POINTS: x , y

               LET z=COMPLEX(x,y)                       !The function for the complex number

               LET w=z^2                               !The definition with changing function

              SET POINT COLOR 4

               PLOT  POINTS:  Re(w),Im(w)               !The show of the point after the change

            end if

        Loop

    END
```
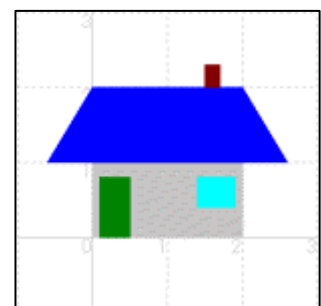
Let's try to execute. Trace a square side skillfully. How is it? The changed figure appeared.
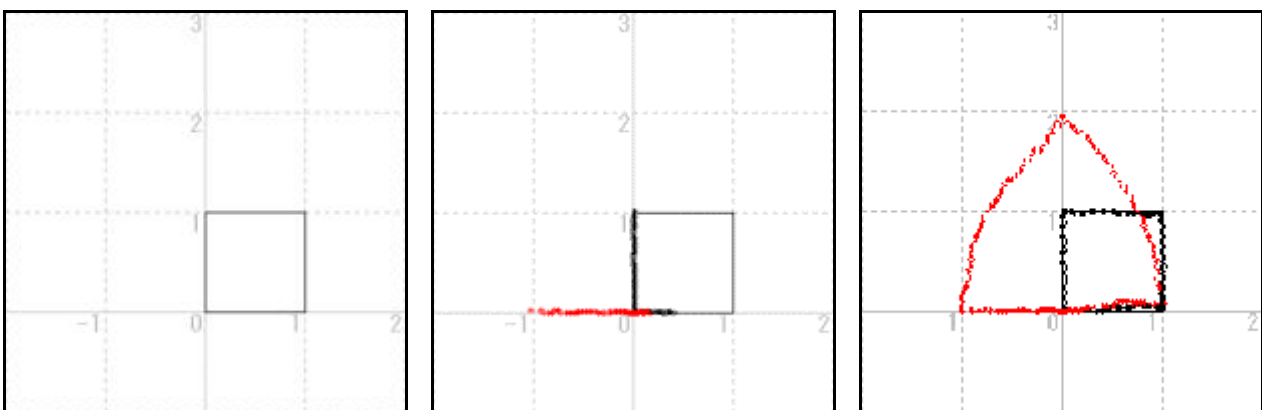
2． The change of $w=z^2$

First, conisder the change of $w=z^2$.

Consider a source figure of a mouse (on the right). Consider how this figure will be changed by the function $w=z^2$.

It is necessary to picture a house first, because there is a " picture definition " in Decimal BASIC. Let's try to define this picture with the name, "House".

```
    PICTURE House
        SET  AREA COLOR 15
        PLOT  AREA:   0,1;  0,0;  2,0;  2,1              ! The wall
        SET  AREA COLOR  2
        PLOT  AREA: -0.6,1;  2.6,1;  2,2;  0,2           ! The  roof
      SET AREA COLOR 10
        PLOT  AREA:  0.1,0;  0.1,0.8;  0.5,0.8;  0.5,0   ! The door
      SET AREA COLOR 5
        PLOT  AREA:  1.4,0.4;  1.9,0.4;  1.9,0.8;  1.4,0.8   ! The window
        SET AREA COLOR 12
        PLOT  AREA:  1.7,2;  1.7,2.3;  1.5,2.3;  1.5,2   ! The chimney
  END PICTURE
```

As previous, let's trace this house with mouse pen. The result is shown below.
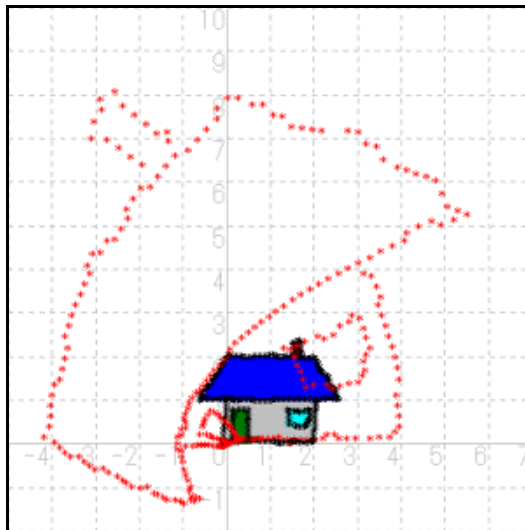
Next, consider a car and an elephant are changed. In some places, they become crushed out of shape.
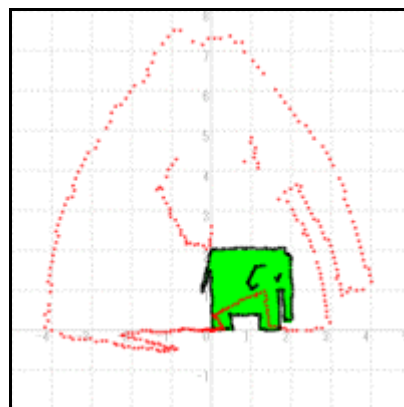
Now, having placed the starting point in the lower left edge of the figure, let's try to shift the position of the figure. In Decimal BASIC, use introduction

. DRAW PICTURE WIDTH SHIFT(*a*,*b*)
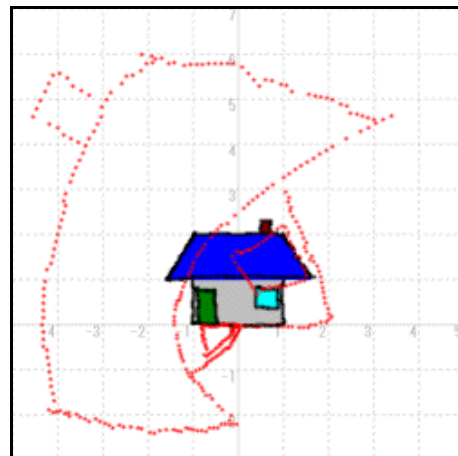
to shift figure.
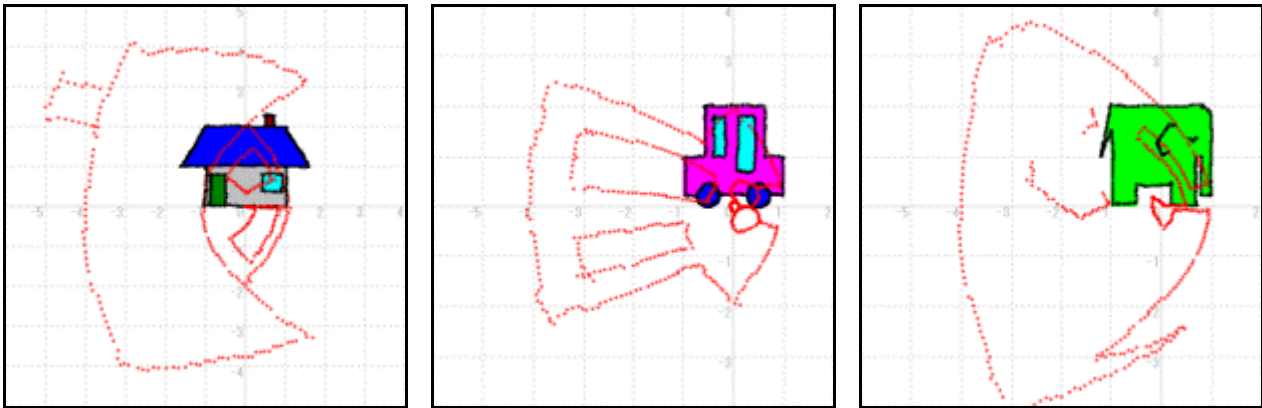
3. The Various othe changes

This time, change the function and see how the figure changes.
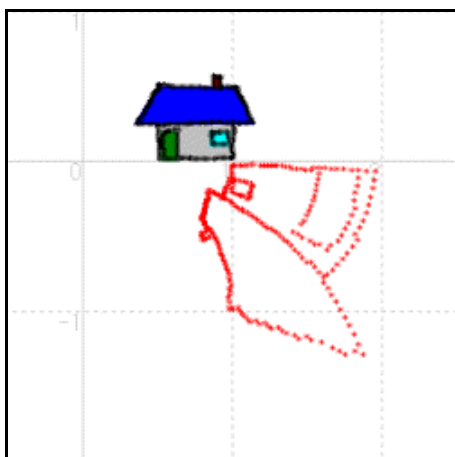
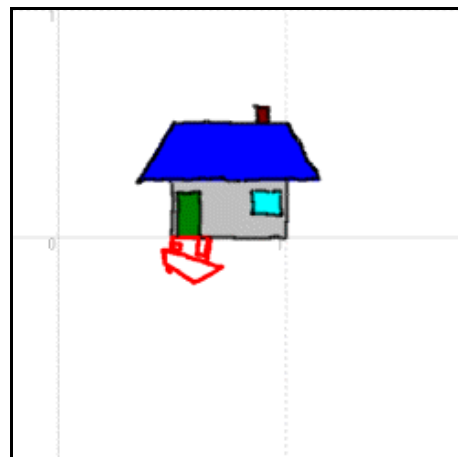For some functions, the figure is changed in a funny way.

The changes of $w=z^2 \quad 2z$
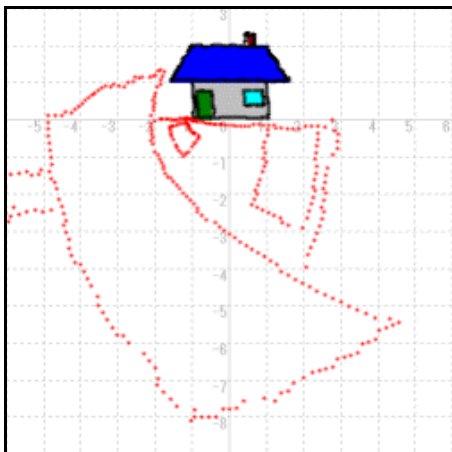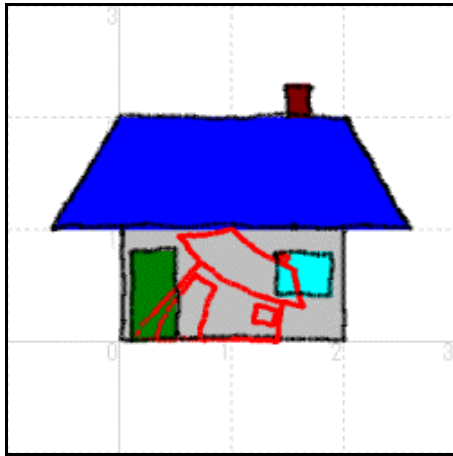

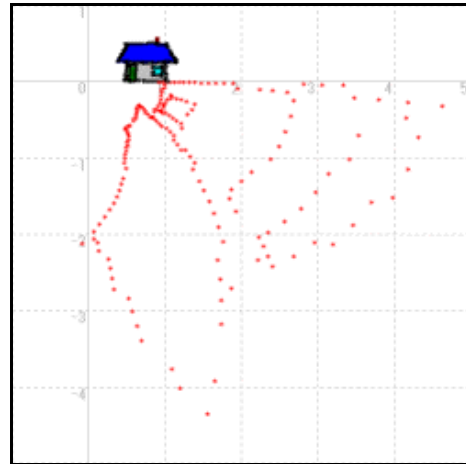
The changes of $w=z^2+z$

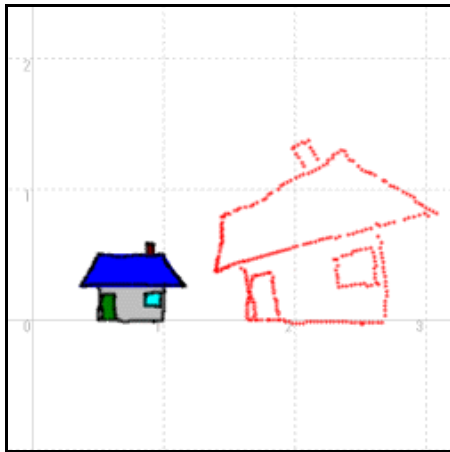The changes of $w=\dfrac{1}{z}$

The changes of $w=\dfrac{1}{z+1}$

The changes of $w=\sqrt{z}$

The changes of $w=\dfrac{1}{z^2}$

The changes of $w=e^z$

The changes of $w=\log z$
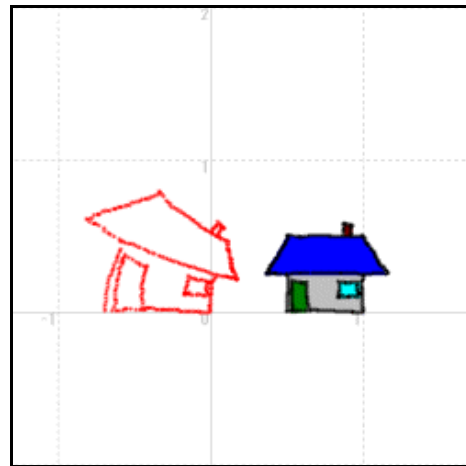
4. The secret of the changes

Next, Consider the secret of the changes. It is possible to make changes a numerical formula.

For details, look at " The Tail of Mobius " by Mr.Nakamura.

For example,consider $w=z^2$.

Let $z=x+yi$.

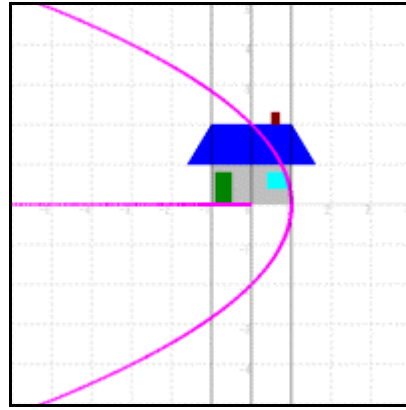Consider how three straight lines, $y=0,1,2$ have been changed in figure 1.

Consider how three straight lines, $x=$ 1, 0, 1,have been changed in the same way in figure 2.

In both next figures, lines are changed into the parabolas which have their focus at the origin. Also there are pseudo-axis in the real plane associated with parabolas.
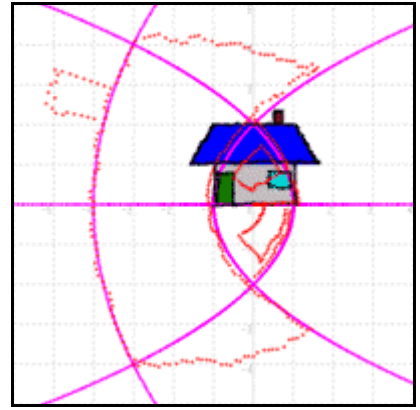
Only two curves appear when changing the three lines of $x=$ 1, 0, 1. This is because the lines x=-1 and a=1 were changed in the same way, so these curves over cap.
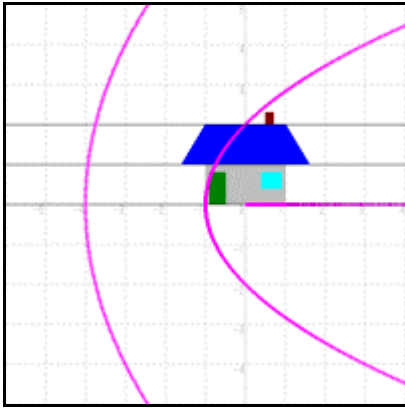
change of a level straight line   change of a perpendicular straight line   synthesizes a changed straight line

How the other changes made? Let's attempt to think of a typical change.

In case of $w = \sqrt{z}$

Like the following figure, the straight line which is parallel to the real axis is shifted to the rectangular hyperbola, the asymptote of which is x axis and y axis.
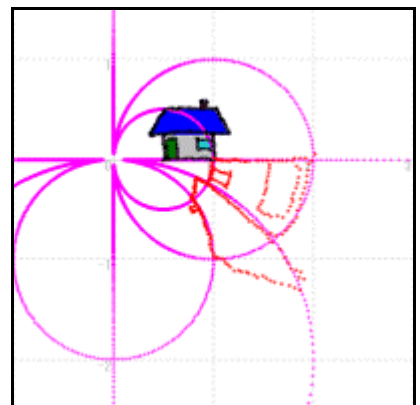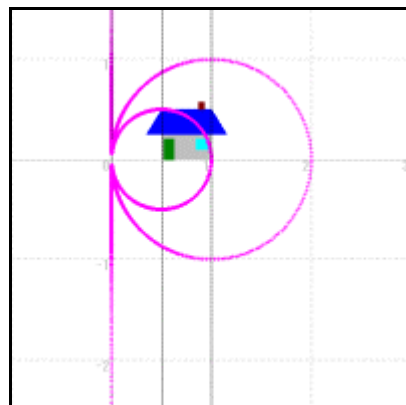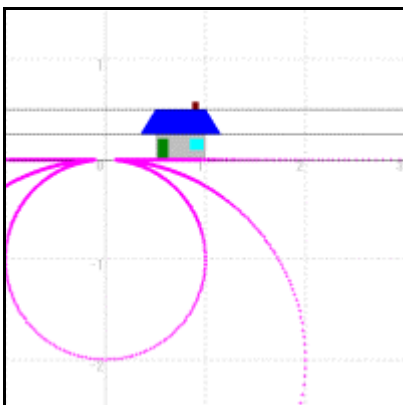
Also, the straight line which is perpendicular to the real axis is shifted to the one in the neighborhood of the origin. In addiotion, it is turned by 45 degrees.
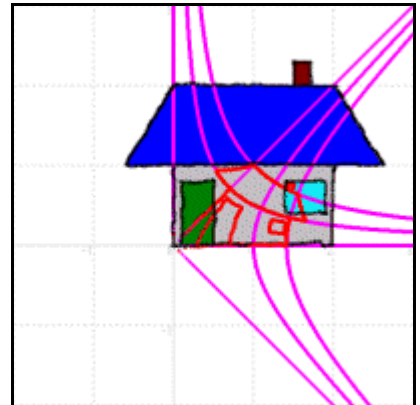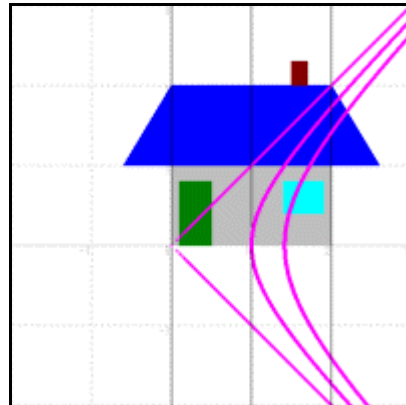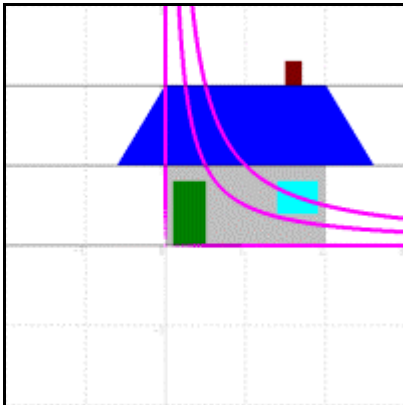
In case of $w = \dfrac{1}{z}$

Like the following figure, the straight line which is parallel to the real axis is shifted to the circle which has a center and a diameter on the y axis.

Also, the straight line which is perpendicular to the real axis is shifted to the circle which has a center and a diameter on the x axis.

In case of $w=e^z$

Like the following figure, the straight line which is parallel to the real axis is shifted to the straight line which passes the origin.

Also, the straight line which is perpendicular to the real axis is shifted to the circl which has a center to the origin.
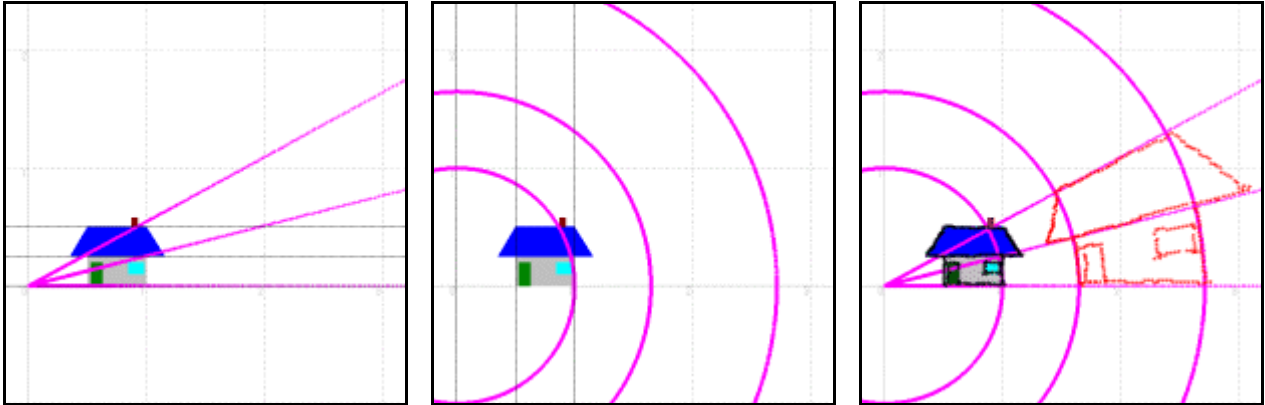

5. All coordinates of the plane changes

We changed various figure in the complex number plane. Next, consider what happens when the whole plane is changed. We try to tranform all straight lines that are parallel and perpendicular to the real axis. This grid figure is changed by the function $w=z^2$.

It becomes a set of parabola which make the origin a focus and there are pseudo-axis int the real plane associated with these parabola.
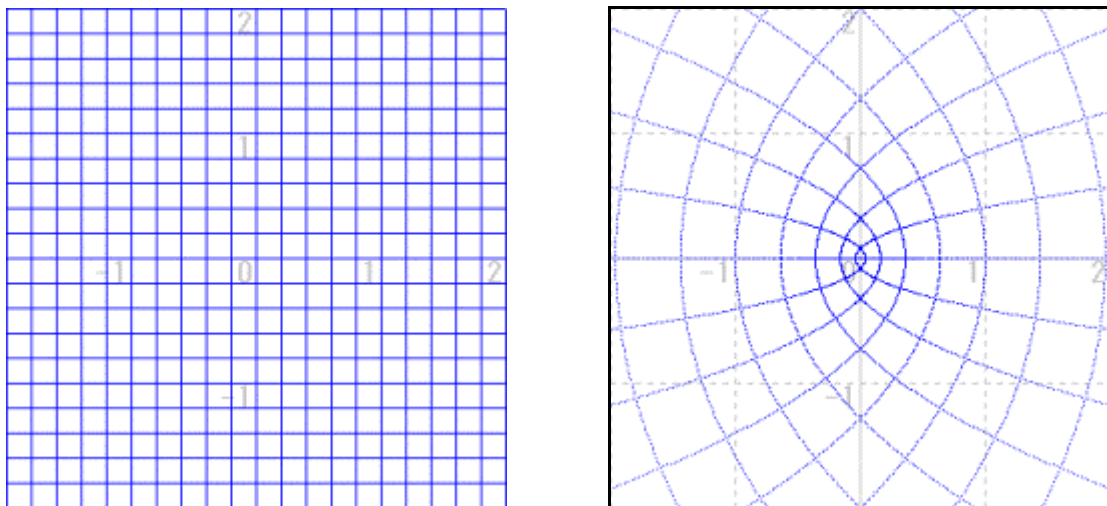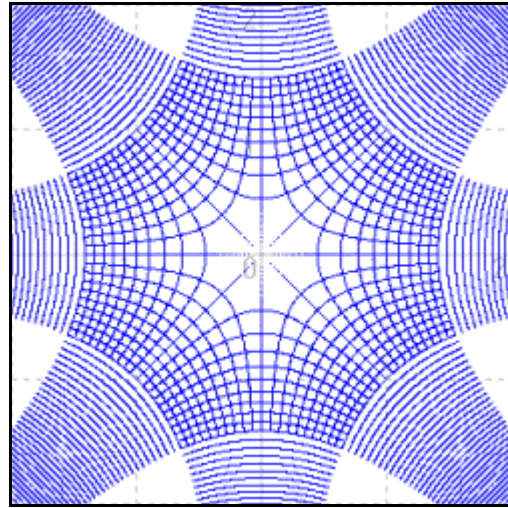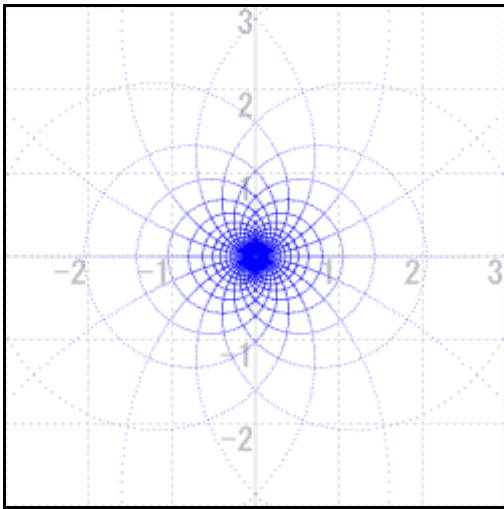

It changes the whole plane in $w=z^2$.

Then, in the other change, it will be how. Let's try to change typical one.
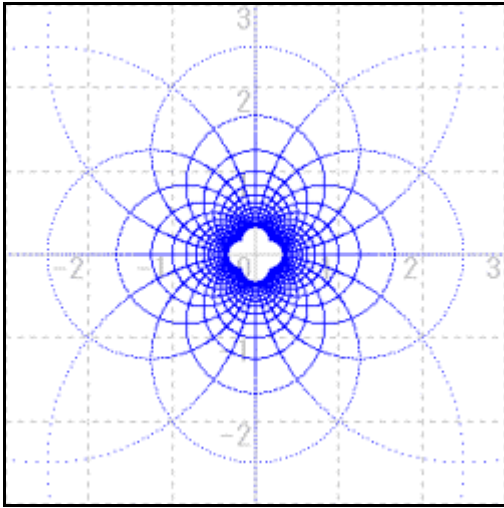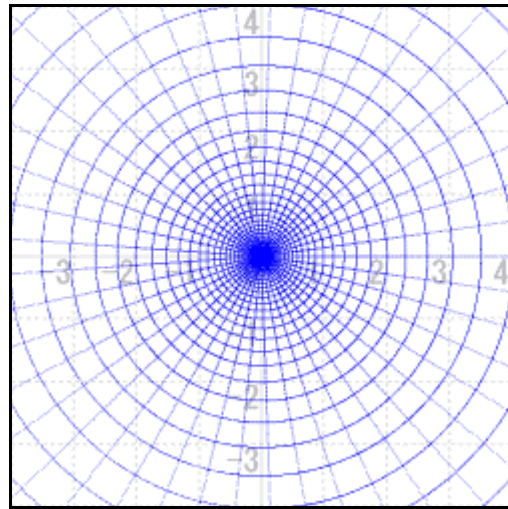
The change of $w = \dfrac{1}{z}$

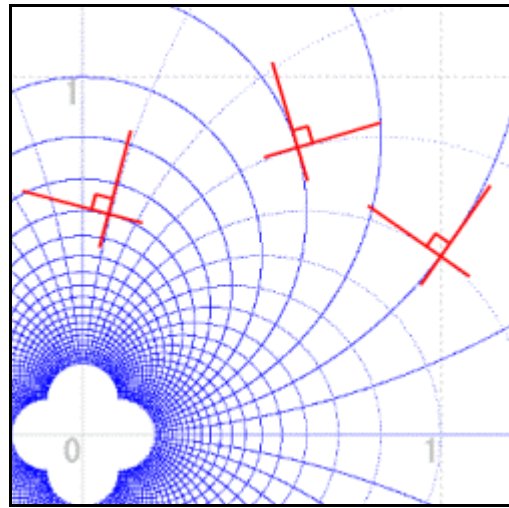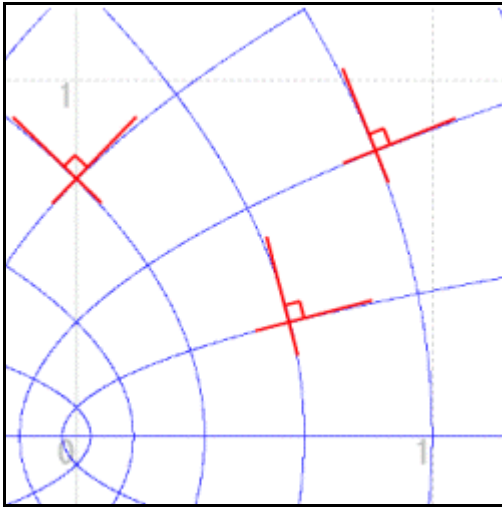The change of $w = \sqrt{z}$

The change of $w = \dfrac{1}{z^2}$

The change of $w = e^z$

The characteristic of these complex changes are that the corner between any insecting perpendicular curves even after both curves have been changed.

The corner made by the intersection of two curves means the corner of the tangent in the point of intersection . Such a change is called " the conformal representation ".

Because it changed rectangular coordinates, the corner of the curve after the change becomes 90 degrees.

However, it is always not the conformal representation in all points. The figure on the right is changed by the function w=z^2. In the origin, the corner formed by the intersection of 2 curves is not 90 degrees. It is 180 degrees.
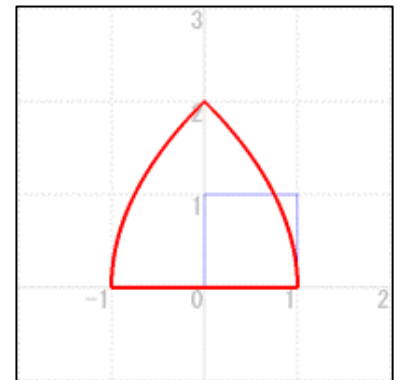

6. The figures in the changed coordinates plane

we attempt to see how a figure will change when changing the whole coordinate plane, and when changing the figure usng a figure,too. From these things, if there is a new coordinate system, the figure can be written there from the beginning, too. Let's attempt to think of changing a figure using function $w=z^2$. The following new coordinates plane was used for this change. First draw a figure at the house like the one on the right.
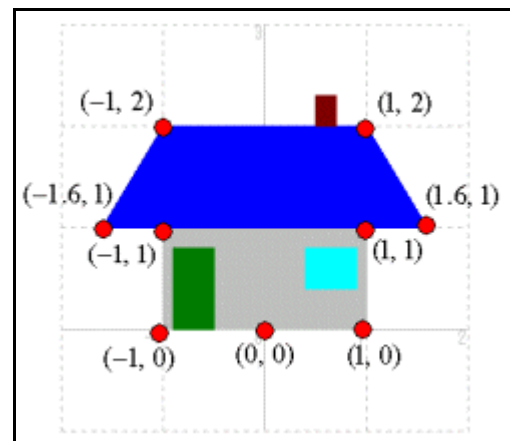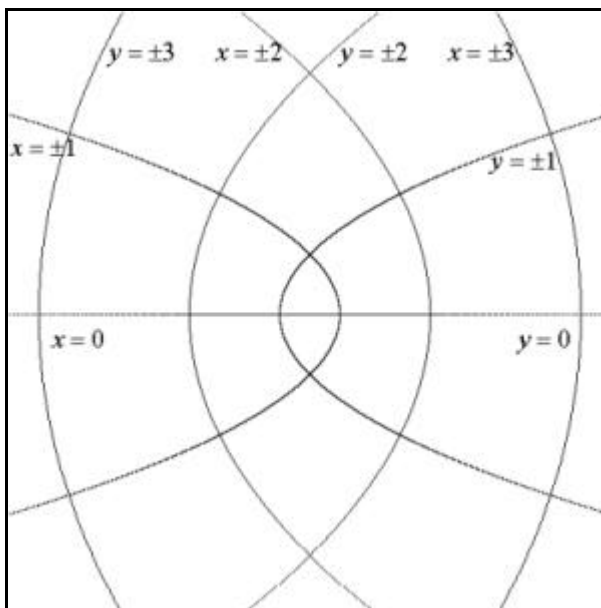
The mouse pen is used to plot the point of the new coordinate system, which correspond to the coordinates of the original points. It is necessary to carefully trace the points with the mouse pen. (For the function $w=z^2$, be careful because the starting point is the same before and after.)
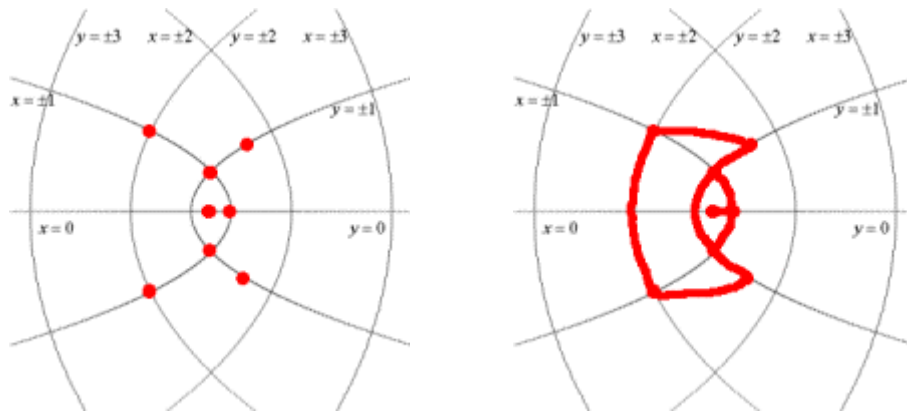
It is in the same way, there is example given about $w=e^z$, too. By changing a coordinate plane base in this way , the figure can be easily pictured.