

自然数のべき和に関する文献集

A list of references on "sums of powers of integers"

ver-1.4

自然数のべき和に関しては、数学的には具象的であって、興味さえあれば高校生でも自分なりの考察が可能であることから、ここでは知る限りで、かつ入手が困難でないと思われる、参考文献の一部、100本余をあげておく。入手困難なものは省いてあるということと、ネット上で調べたものがほとんどであるから全てを網羅していないことに留意いただきたい。またべき和に直接的でない周辺の話題に関するものも含まれている。書籍関係は一部にその関連記述がある程度のものが多いため、ごく一部しかとりあげていないこともはじめにお断わりしておく。^{1,2}

参考文献

- [1] D.Acu, Some algorithm for the sums of integer powers, Mathematics Magazine, 61(1988), 189-191
- [2] J.Aczel, General solution of a system of functional equations satisfied by sums of powers on arithmetic progressions, Aequationes Math. 21(1980), 39-43
- [3] D.Allison, A note on sums of powers of integers, American Mathematical Monthly, 68(1961), 272
- [4] I.Anderson, Sums of squares and binomial coefficients, Mathematical Gazette, 65(1981), 87-92
- [5] O.D.Anderson, Explicit formula for summing $1^k + 2^k + \dots + n^k$, Mathematics and Computer Education, 24(1990), 225-231
- [6] O.D.Anderson, Summing powers of integers, Mathematical Spectrum, 23(1990/1991), 116-121
- [7] A.B.Ayoub, A note on the sums of squares of natural numbers, Mathematics and Computer Education, 26(1992), 246-247
- [8] A.F.Beardon, Sums of powers of integers, American Mathematical Monthly 103(1996), 201-213
- [9] A.Benjamin and M.E.Orrison, Two Quick Combinatorial Proofs of $\sum_{k=1}^n k^3 = \binom{n+1}{2}^2$, The College Mathematics Journal, Vol 33, No. 5, pp. 406-408
- [10] D.M.Bloom, An old algorithm for the sum of integer powers, Mathematics Magazine, 66(1993), 304-305
- [11] C.B.Boyer, Pascal's formula for the sums of powers of the integers, Scripta Math, 9(1943), 237-244
- [12] R.Brawer, and M.Pirovino The linear algebra of the Pascal matrix, Linear Algebra Appl, 174(1992), 13-23

¹このリストは「自然数のべき和に関するメモ II」のものに加筆したもの。

²資料に不備があればゴメンナサイ !!

- [13] G.F.C.de Bruyn and J.M.de Villiers, Formulas for $1^p + 2^p + 3^p + \dots + n^p$, Fibonacci Quarterly, 32.3(1994), 271-276
- [14] B.L.Burrows and R.F.Talbot, Sums of powers of integers, American Mathematical Monthly 91(1984), 394-403
- [15] S.R.Cavior, A theorem on power sums, Fibonacci Quarterly, 6.2(1968), 157-161
- [16] F.Chorlton, Finite sums of powers of the natural numbers, Mathematical Gazette, 82(1998), 95-96
- [17] J.H.Conway and R.K.Guy, The book of numbers, 1996, Springer-Verlag ISBN0-387-97993-X
- [18] E.Creutz, Sums of integral powers of consecutive integers, Nuclear Science and Engineering, 52(1973), 142-144
- [19] T.Crilly and S.Millward, Sums of powers of integers-a general method, Mathematical Gazette, 72(1988), 205-207
- [20] A.Cupillari, Proof without words: $1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \frac{[n(n+1)]^2}{4}$, Mathematics Magazine, 62(1989), 259
- [21] D.Desbrow, Sums of integer powers, Mathematical Gazette, 66(1982), 97-100
- [22] D.Desbrow, Volumetric proof of the sum of squares formula, Mathematical Gazette, 83(1999), 256-257
- [23] J.Ding and T.H.Fay, Bernoulli numbers and calculating the sums $1^k + 2^k + \dots + n^k$, Mathematics and Computer Education, 30(1996), 70-79
- [24] J.Doucet and A.Saleh-Jahromi, Sums of powers of integers, Research paper in Proceedings of the Louisiana-Mississippi Section of the Mathematical Association of America, Spring 2002 (On-Line)
<http://www.mc.edu/campus/users/travis/maa/proceedings/spring2002/doucet.jahroni.pdf>
- [25] S.M.Edmonds, Sums of powers of natural numbers, Mathematical Gazette, 41(1957), 187-189
- [26] A.W.F Edwards, Sums of powers of integers : a little of the history, Mathematical Gazette, 66(1982), 22-28
- [27] A.W.F Edwards, A quick route to sums of powers, American Mathematical Monthly 93(1986), 451-455
- [28] T.H.Fay, A note on the sums $1^k + 2^k + \dots + n^k$, Mathematics and Computer Education, 28(1994), 46-47
- [29] T.H.Fay, Remarks on the sums of powers of integers, Mathematics and Computer Education, 30(1996), 174-178
- [30] T.H.Fay and K.R.S.Sastry, A further note on the sums $1^k + 2^k + \dots + n^k$, Mathematics and Computer Education, 29(1995), 253-261

- [31] T.H.Fay and K.R.S.Sastry, Sums of powers of an arithmetic progression, *Mathematical Spectrum*, 30(1997/1998), 10-12
- [32] T.H.Fay and B.L.Piazza A combinatorial approach to the calculation of $1^k + 2^k + \cdots + n^k$, *Mathematics and Computer Education*, 29(1995), 269-278
- [33] N.Gauthier, Sum of the m -th powers of n successive terms of an arithmetic sequence: $b^m + (a + b)^m + (2a + b)^m + \cdots + ((n - 1)a + b)^m$, *Int. J. Math. Educ. Sci. Technol.*, 37.2(2006), 207-215
- [34] I.Gessel, A formula for power sums, *American Mathematical Monthly*, 95(1988), 961-962
- [35] I.M.Gessel and X.G.Viennot, Determinants, paths, and plane partitions, preprint, (1989)
- [36] R.L.Graham, D.E.Knuth and O, Patashnik, *Concrete mathematics*, 1994, Addison-Wesley, ISBN 0-201-55802-5
- [37] J.Grabner and H.Prodinger, Some identities for Chebyshev polynomials, *Portugalia Math*, 59(2002), 311-314
- [38] C.V.Gregg, Realations between the sums of numbers of the natural numbers, *Mathematical Gazette*, 44(1960), 118-120
- [39] M.Griffiths, Sums of powers of the terms in any finite arithmetic progression, *Mathematical Gazette*, 86(2002), 269-270
- [40] S.-L.Guo and F.Qi, Recursion formulae for $\sum_{m=1}^n m^k$, *Z.Anal.Anwendungen(J.Anal.Appl)*, 18(1999).4, 1123-1130
- [41] V.J.W.Guo and J.Zeng, A q -Analogue of Faulhaber 's Formula for Sums of Powers, *Electron.J.Combin* 11(2) (2005), #R19
- [42] S.L.Gupta An identity involving the sum of the k th powers of the first n natural numbers, *Mathematical Gazette*, 56(1972), 128-129
- [43] E.Hairer and G.Wanner, *Analysis by its history*, UTM, 1996, Springer-Verlag, ISBN0-387-94551-2
- [44] V.E.Hoggatt, Jr, A note on the summation of squares, *Fibonacci Quarterly*, 15.4 (1977) 367-369
- [45] V.E.Hoggatt, Jr, Corrections to "A note on the summation of squares", *Fibonacci Quarterly*, 18.1 (1980) 82-83
- [46] P.Holmes, Proof without words: $\sum_{r=1}^n r^3 = (\sum_{r=1}^n r)^2$, *Mathematical Gazette*, 86(2002), 267-268
- [47] F.T.Howard, Sums of powers of integers, *Mathematical Spectrum*, 26(1993/1994), 103-109
- [48] F.T.Howard, Sums of powers of integer via generating functions, *Fibonacci Quarterly*, 34.3(1996), 244-256
- [49] F.T.Howard, Lacunary recurrences for sums of powers of integers, *Fibonacci Quarterly*, 36.5(1998), 435-442

- [50] Y.Inaba, A note on the sums $1^k + (1+d)^k + (1+2d)^k + \dots + (1+nd)^k$, *Int. J. Math. Educ. Sci. Technol*, 35.3(2004), 440-445
- [51] Y.Inaba, Hyper-Sums of Powers of Integers and the Akiyama-Tanigawa Matrix, *J. Integer Seq*, Article 05.2.8
- [52] S.Jafari, Summing the series $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ using Pascal's identity, *Mathematical Spectrum*, 26(1993/1994), 50-51
- [53] L.Jianjin, On the development of the formulae for sums of powers of integers, *Historia Sci*, 13-2(2003), 101-110
- [54] V.Kac and P.Cheung, *Quantum calculus*, Universitext, 2000, Springer-Verlag
- [55] R.A.Kahn, A simple derivation of a Formula for $\sum_{k=1}^n k^r$, *Fibonacci Quarterly*, 19.2 (1981), 177-180
- [56] D.Kalman, Sums of powers by matrix methods, *Fibonacci Quarterly*, 28.1(1990), 60-71
- [57] PL.Kannappan, Sum of powers of integers and the additive Cauchy equation, *Soochow Journal of Mathematics*, 27(2001), 89-95
- [58] C.Kelly, An algorithm for sums of integer powers, *Mathematics Magazine*, 57(1984), 296-297
- [59] T.Kim, Sums of powers of consecutive q -integers, arXiv:math/0501531v1 [math.NT] 29 Jan 2005
- [60] D.E.Knuth, Johann Faulhaber and sums of powers, *Mathematics of Computation*, 61(1993), 277-294
- [61] T.Koshy, Summing integer cubes using Thébault's array of arithmetic sequences, *Mathematical Gazette*, 86(2002), 271-272
- [62] T.C.T.Kotiah, Sums of powers of integers-A review, *Int.J.Math.Educ.Sci.Technol.*, 24-6(1993), 863-874
- [63] K.R.Kundert, Sums of powers as sums of products, *Mathematics Magazine*, 54(1981), 81-83
- [64] L.S.Levy, Summation of the series $1^n + 2^n + \dots + x^n$ using elementary calculus, *American Mathematical Monthly*, 77(1970), 840-847
- [65] C.L.Liu, *Intoroduction to combinatorial mathematics*, 1968, McGraw-Hill, ISBN0-07-038124-0
- [66] J.Luo, On the development of the formulae for sums of powers of integers. *Historia Sci*. (2)13(2003), no. 2, 101-110
- [67] L.A.Lyusternik and A.R.Yanpol'skii, eds, *Mathematical Analysis-Functions, Limits, Series, Continued Fractions*, 1965, Pergmon Press
- [68] G.Mackiw, A combinatorial approach to sums of integer powers, *Mathematics Magazine*, 73(2000), 44-46

- [69] R.S.Melham, On sums of powers of terms in a linear recurrence, *Portugalia Math*, 56(1999), 501-508
- [70] L.Monk, D.Tang and D.Brown, Identities for generalized Fibonacci Numbers, *Int. J. Math. Educ. Sci. Technol*, 35.3(2004), 436-439
- [71] W.O.J.Moser, Sums of d th powers, *Mathematical Gazette*, 75(1991), 332-334
- [72] W.O.J.Moser, Sum of powers of a finite sequence:a geometric approach, *Crux Mathematicorum*, 24(1998), 145-147
- [73] J.I.Naus, An instructive derivation of sums of powers and factorial powers of integers, *Amer. Statist*, 20(1966), 42-43
- [74] K.Nishimoto and H.M.Srivastava, Evaluation of sum of powers of natural numbers by Means of fractional calculus, *J.College Engrg.Nihon Univ.Ser.B32*(1991), 127-132
- [75] J.Nunemacher and R.Young, On the sum of consecutive k -th powers, *Mathematics Magazine*, 60(1987), 237-238
- [76] R.W.Owens, Sums of powers of integers, *Mathematics Magazine*, 65(1992), 38-40
- [77] R.V.Parker, Sums of powers of the integers, *Mathematical Gazette*, 42(1958), 91-95
- [78] J.L.Paul, On the sum of the k th powers of the first n integers, *American Mathematical Monthly* 78(1971), 271-272
- [79] D.E.Penney and C.Pomerance, Multiplicative relations for sums of initial k th powers, *American Mathematical Monthly* 92(1985), 729-731
- [80] T.J.Pfaff, Deriving a Formula for Sums of Powers of Integers. *The PME Journal*, 12.7(2007)
- [81] M.S.Phatak, Sums of powers of natural numbers, *Indian J.pure appl.Math.*, 21(10)(1990), 879-887
- [82] P.A.Piza, Powers of sums and sums of powers, *Mathematics Magazine*, 25(1952), 137-142
- [83] J.Riordan, Generating functions for powers of Fibonacci numbers, *Duke Math.J.* 29(1962), 5-12.
- [84] J.Riordan, *Introduction to combinatorial analysis*, 2002, Dover, ISBN0-486-42536-3(Original printed, 1958, John Wiley)
- [85] A.Robertson and T.J.Osler, Euler's little summation formula and sums of powers, *Mathematical Spectrum*, to appear
- [86] H.J.Schultz, The sums of the k -th powers of the first n integers, *American Mathematical Monthly* 87(1980), 478-481
- [87] J.A.Scott, On sums of powers of the natural numbers, *Mathematical Gazette*, 85(2001), 89-90
- [88] M.J.A.Sharkey, An identity involving the sums of powers, *Mathematical Gazette*, 57(1973), 131-133

- [89] S.A.Shirali, On Sums of Powers of Integers, Resonance, 12.7(2007), 27-43
- [90] D.R.Snow, Formulas for sums of powers of integers by functional equations, Aeq. Math. 18(1978), 269-285.
- [91] M.Z.Spivey, The Euler-Maclaurin Formula and Sums of Powers, Mathematics Magazine, 79(2006), 61-65
- [92] H.M.Srivastava, J.M.C.Joshi and C.S.Bisht, Fractional calculus and the sum of powers of natural numbers, Stud. Appl. Math. 85(1991), 183-193.
- [93] R.P.Stanley, Enumerative Combinatorics, Vol.1, 1997, CAMBRIDGE ADVANCED VOL.49, ISBN0-521-55309-1
- [94] R.P.Stanley, Enumerative Combinatorics, Vol.2, 1999, CAMBRIDGE ADVANCED VOL.62, ISBN0-521-56069-1
- [95] J.A.Suzuki, Intoroducing the sums of powers, The College Mathematics Journal, 35(2004), 303-305
- [96] L.I.Szabó, Some equations concerning the sums of powers of integers, Acta Sci.Math. (Szeged) 67(2001), 501-503
- [97] M.Tepper, Combinations and sums of powers, Fibonacci Quarterly, 12.2(1974), 196-198
- [98] M.Tepper, Sums of powers, Mathematics Magazine, 38(1965), 17-19
- [99] B.Turner, Sums of powers of integers via the binomial theorem, Mathematics Magazine, 53(1980), 92-96
- [100] R.Venkatachalam, U.C.Sharma, An engineer's approach to a general algorithm for finding the sum of powers of natural numbers, Int. J. of Mechanical Engineering Educ, 31.1(2003), 1-14
- [101] C.Wagner, Combinatorial proofs of formulas for power sums, Arch.Math.68(1997), 464-467
- [102] S.O.Warnaar, On the q -analogue of the sum of cubes, Electron. J. Combin. 11 (2004), #N13
- [103] N.Wermuth, Proof without words: factorising sums of integers taken to a small fixed power, (2002), <http://psystat.sowi.uni-mainz.de/wermuth/pdfs/sumint.pdf>
- [104] J.Wiener, A calculus exercise for the sums of integer powers, Mathematics Magazine, 65(1992), 249-251
- [105] The On-Line Encyclopedia of Integer Sequences (On-Line)
<http://www.research.att.com/njas/sequences/>
- [106] Eric W. Weisstein, world of MATHEMATICS (On-Line)
<http://mathworld.wolfram.com/>