四分円内の円と正三角形と正方形Ⅰ，Ⅱ

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半径1の四分円内に，図のように直径 a の円，1辺 a の正三角形と正方形を配置して a の値を求めてみた。

Ⅰ　円，正方形，正三角形が四分円の半径に並ぶとき（29通り）。

Ⅱ　円，正方形，正三角形が四分円の中心角の二等分線に関して対称に並ぶとき（24通り）。

Ⅲ　円，正方形，正三角形が四分円の円弧に並ぶとき（13通り）。

今回は，Ⅰ，Ⅱの場合である。

■：正方形は四分円の中心角の二等分線に関して対称である。

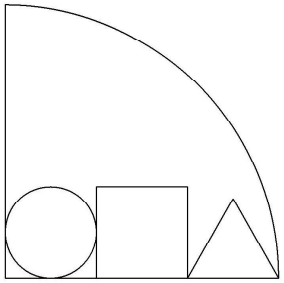
△：正三角形の一辺は四分円の半径に垂直である。

▲：正三角形は四分円の中心角の二等分線に関して対称である。

また，Ⅰの(7)，(8)，(10)，(14)は a の満たす方程式が高次方程式となり，求めることができなかった。

Ⅰ

(1)



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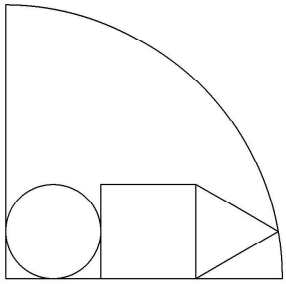

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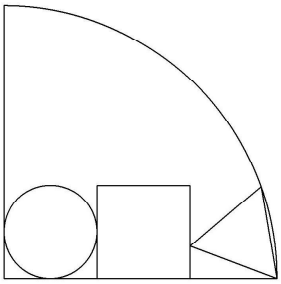

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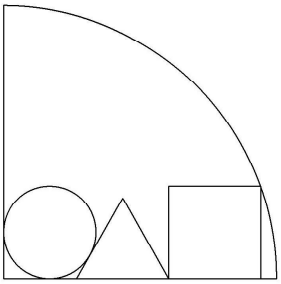

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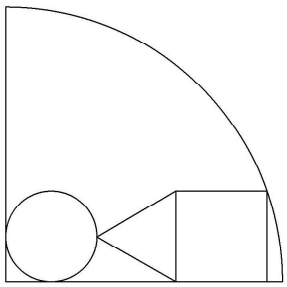

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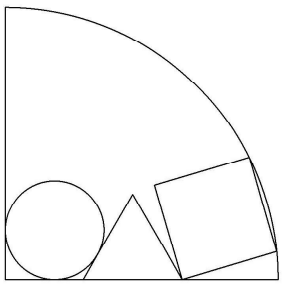

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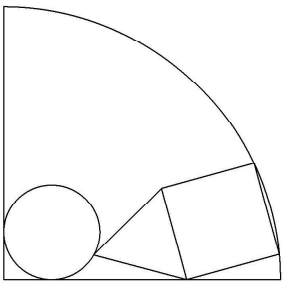

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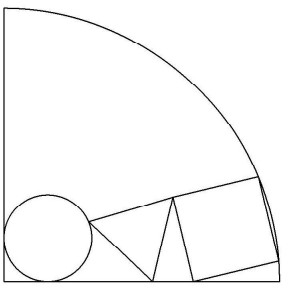

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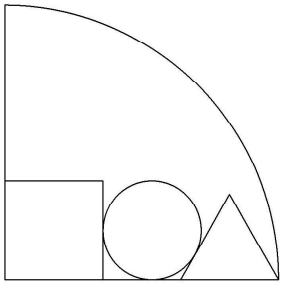

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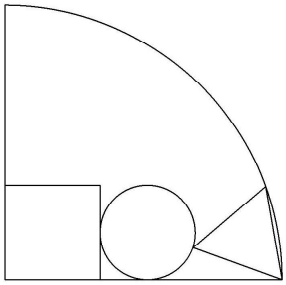

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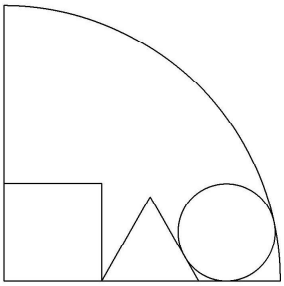

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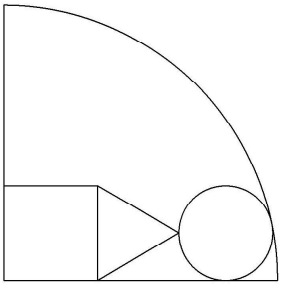

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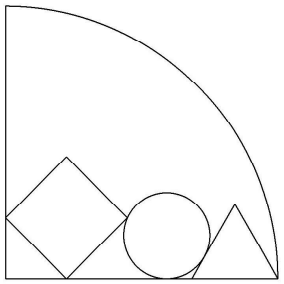

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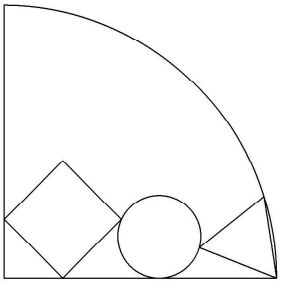

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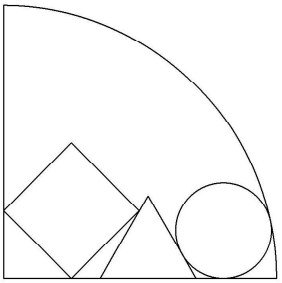

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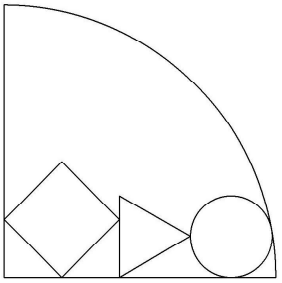

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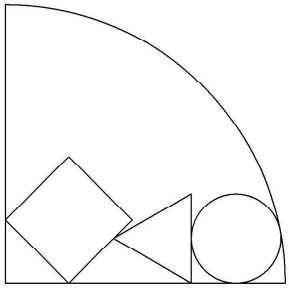

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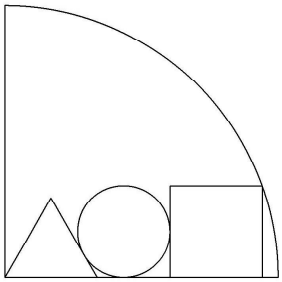

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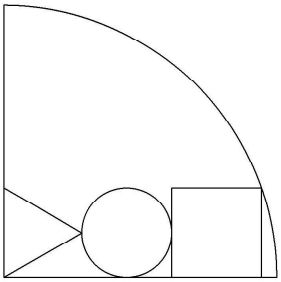

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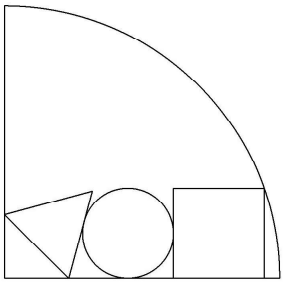

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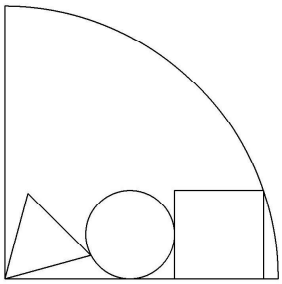

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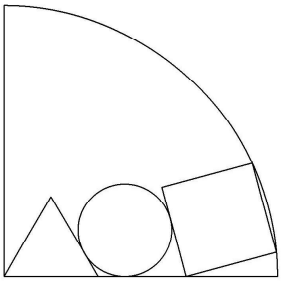

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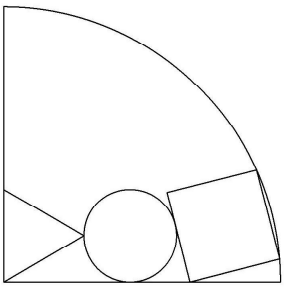

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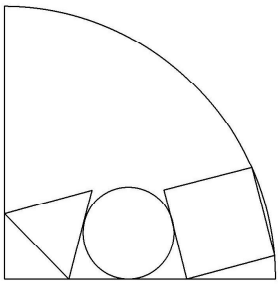

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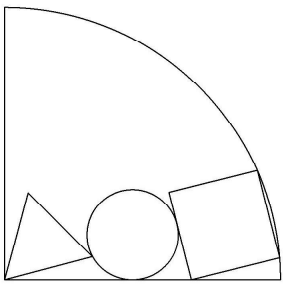

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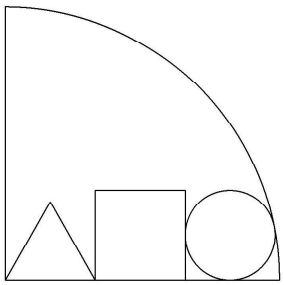

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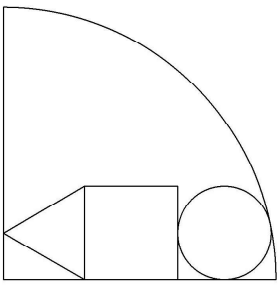

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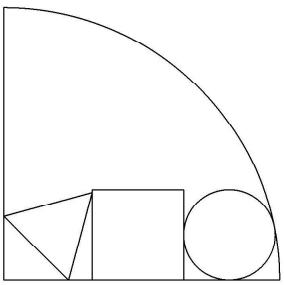

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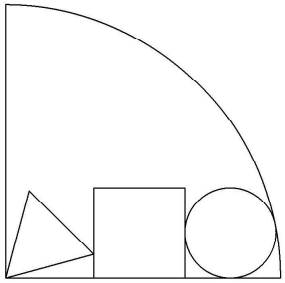
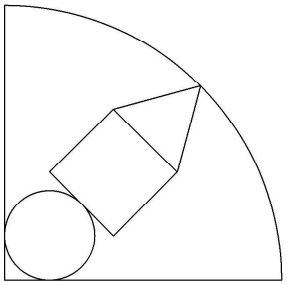


Ⅱ

(1)



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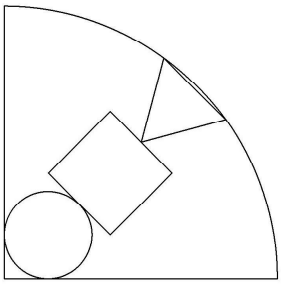

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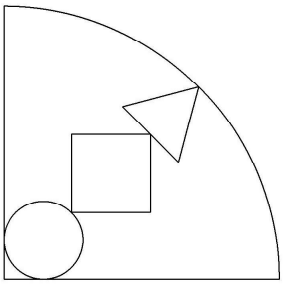

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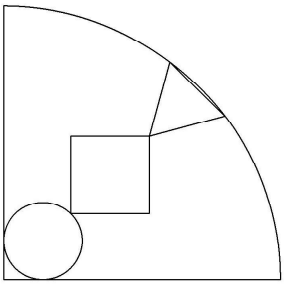

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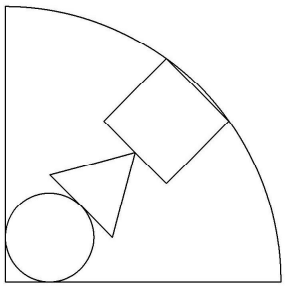

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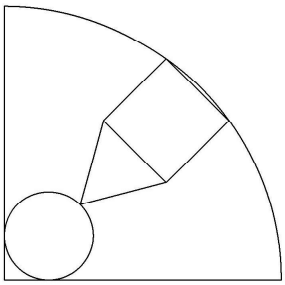

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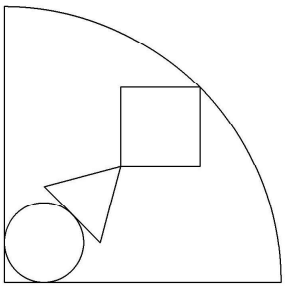

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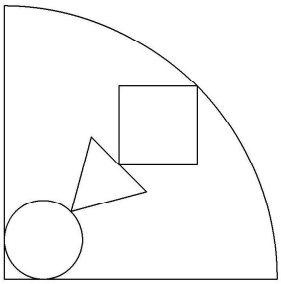

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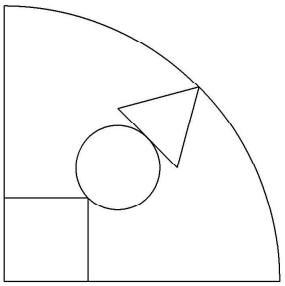

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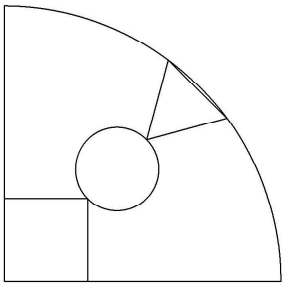

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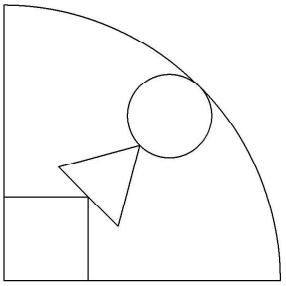

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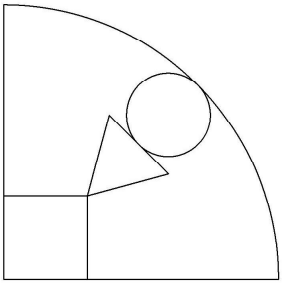

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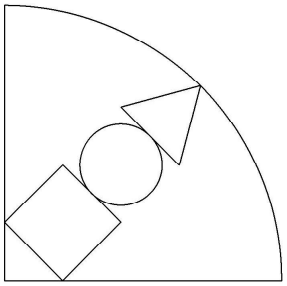

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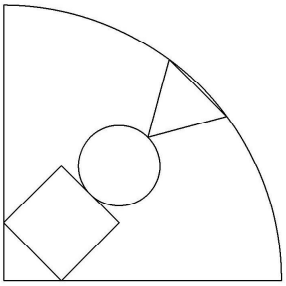

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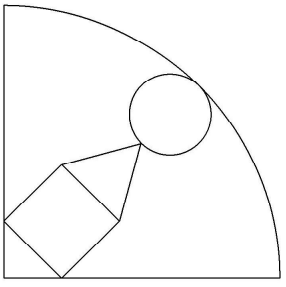

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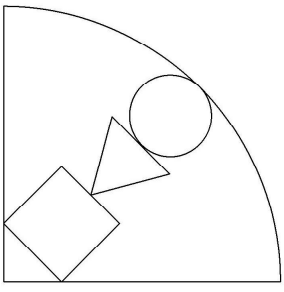

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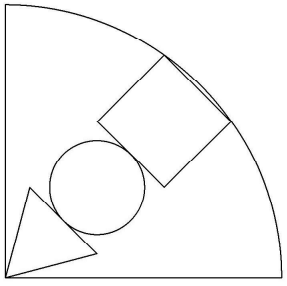

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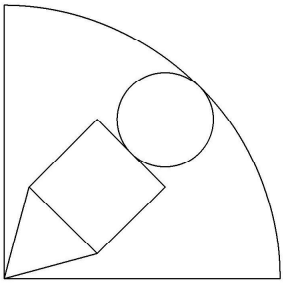

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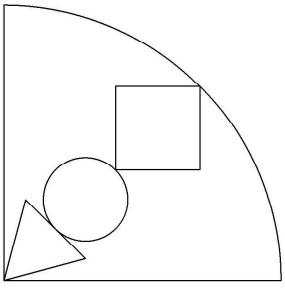

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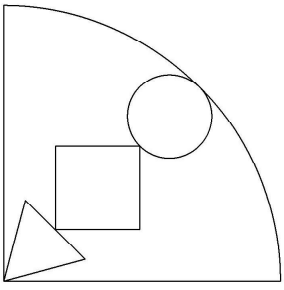

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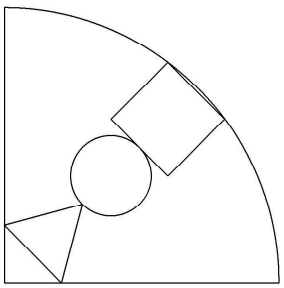

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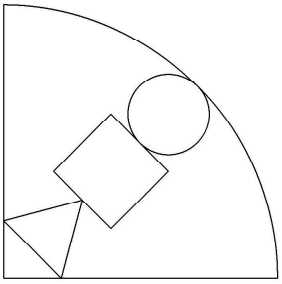

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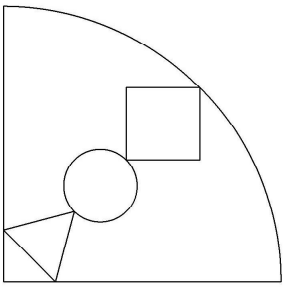

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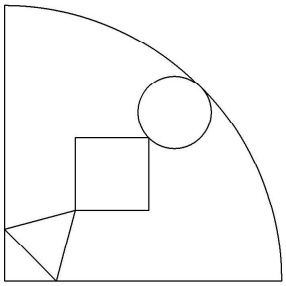

答

I

(1) $a=\dfrac{1}{3}$

(2) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$ $(\fallingdotseq 0.343724)$

(3) $a^4+4a^3+11a^2-16a+4=0$ の解で，$k=\dfrac{\sqrt[3]{7235+36\sqrt{4089}}+\sqrt[3]{7235-36\sqrt{4089}}+5}{6}$ とおくと，

$a=\dfrac{1}{2}\left(\sqrt{2k-5}-\sqrt{-2k-5+\dfrac{60}{\sqrt{2k-5}}}-2\right)$ $(\fallingdotseq 0.340313)$

(4) $a=\sqrt{\dfrac{6(44-5\sqrt{3})}{1861}}$ $(\fallingdotseq 0.337547)$

(5) $a=2\sqrt{\dfrac{23-8\sqrt{3}}{337}}$ $(\fallingdotseq 0.329438)$

(6) $a=\sqrt{\dfrac{6\{2108-579\sqrt{3}-\sqrt{3}(1426276-666216\sqrt{3})\}}{9217}}$ $(\fallingdotseq 0.361927)$

(7) $-256(-26405+15244\sqrt{3})+256(-107527+62064\sqrt{3})a^2-32(-1095219+631484\sqrt{3})a^4+$
$16(-982989+562616\sqrt{3})a^6+(4699805-2580508\sqrt{3})a^8+2(-449445+178664\sqrt{3})a^{10}+170834a^{12}=0$
$(a\fallingdotseq 0.347267)$

(8) $256-256(17+4\sqrt{3})a^2+32(1055+424\sqrt{3})a^4-16(9545+4724\sqrt{3})a^6+(406953+216496\sqrt{3})a^8$
$-4(134895+72242\sqrt{3})a^{10}+8(46643+24622\sqrt{3})a^{12}-72(1767+926\sqrt{3})a^{14}+4(4097+2152\sqrt{3})a^{16}=0$
$(a\fallingdotseq 0.318938)$

(9) $a=\dfrac{15-\sqrt{3}}{37}$ $(a\fallingdotseq 0.358593)$

(10) $16-96a+184a^2-8(12+\sqrt{3})a^3-51a^4+2(15+13\sqrt{3})a^5+40a^6=0$ $(a\fallingdotseq 0.34317)$

(11) $a=\dfrac{-6(49-8\sqrt{3})+4\sqrt{3(29005-5006\sqrt{3})}}{2209}$ $(\fallingdotseq 0.351784)$

(12) $a=\dfrac{-2+\sqrt{3}+\sqrt{19-10\sqrt{3}}}{3}$ $(\fallingdotseq 0.342668)$

(13) $a=\dfrac{2}{1320001}(59070+162138\sqrt{2}+18883\sqrt{3}-28482\sqrt{6}$
$-3\sqrt{2}\sqrt{-7314893929+5302933885\sqrt{2}-1387170392\sqrt{3}+946127280\sqrt{6}})$ $(\fallingdotseq 0.316658)$

(14) $16(121+36\sqrt{2})-128(72+121\sqrt{2})a+32(3195+851\sqrt{2})a^2$

$-16(4152+10994\sqrt{2}+121\sqrt{3}+36\sqrt{6})a^3+16(20194+133\sqrt{2}+288\sqrt{3}+48\sqrt{6})a^4$

$-8(-19348+16350\sqrt{2}+1449\sqrt{3}+929\sqrt{6})a^5-4(2089+35474\sqrt{2}-2848\sqrt{3}+5924\sqrt{6})a^6$

$-8(-7364-996\sqrt{2}-9966\sqrt{3}+321\sqrt{6})a^7-4(-21576-4129\sqrt{2}+3168\sqrt{3}+5324\sqrt{6})a^8$

$+8(-1376-4990\sqrt{2}-4541\sqrt{3}+1238\sqrt{6})a^9+(-20637+8996\sqrt{2}+2384\sqrt{3}+6684\sqrt{6})a^{10}$

$+2(-1540+6114\sqrt{2}+7511\sqrt{3}+741\sqrt{6})a^{11}+12049a^{12}=0$ （$a\fallingdotseq 0.304689$）

(15) $a=\dfrac{2}{69169}\{-3(19695-13222\sqrt{2}+9764\sqrt{3}-6864\sqrt{6})+$

$\sqrt{6(2640486213-1841678993\sqrt{2}+1459123846\sqrt{3}-1030246272\sqrt{6})}\}$（$\fallingdotseq 0.351064$）

(16) $a=-54+38\sqrt{2}-31\sqrt{3}+22\sqrt{6}+\sqrt{11699-8272\sqrt{2}+6754\sqrt{3}-4776\sqrt{6}}$ （$\fallingdotseq 0.300766$）

(17) $a=\dfrac{-2(177-52\sqrt{2}-68\sqrt{3}+14\sqrt{6})+2\sqrt{2}\sqrt{72709-25814\sqrt{2}-31478\sqrt{3}+9717\sqrt{6}}}{529}$ （$\fallingdotseq 0.320383$）

(18) $a=\sqrt{\dfrac{6(44-5\sqrt{3})}{1861}}$ （$\fallingdotseq 0.337547$）

(19) $a=2\sqrt{\dfrac{23-8\sqrt{3}}{337}}$ （$\fallingdotseq 0.329438$）

(20) $a=\sqrt{\dfrac{1571-573\sqrt{2}+565\sqrt{3}-475\sqrt{6}}{5281}}$ （$\fallingdotseq 0.330188$）

(21) $a=2\sqrt{\dfrac{2(61381909-28595134\sqrt{2}+17563922\sqrt{3}-8314544\sqrt{6}+c)}{822183073}}$ （$\fallingdotseq 0.325598$）

ただし，$c=2\sqrt{-789207640885927+267735339142723\sqrt{2}-216983381956407\sqrt{3}+329697216371743\sqrt{6}}$

(22) $a=\sqrt{\dfrac{\sqrt[3]{e+f}+\sqrt[3]{e-f}+45554+5666\sqrt{3}}{119631}}$ （$\fallingdotseq 0.342796$）

ただし，$e=-309842980073407-143647319680593\sqrt{3}$，

$f=1076679\sqrt{3(134765452358427047+74051166188382620\sqrt{3})}$

(23) $a=2\sqrt{\dfrac{\sqrt[3]{e+f}+\sqrt[3]{e-f}+42803+1836\sqrt{3}}{336387}}$ （$\fallingdotseq 0.334497$）

ただし，$e=-2(73713397474337+42188503703184\sqrt{3})$，

$f=672774\sqrt{3(75199779519507107+43415695006678208\sqrt{3})}$

(24) $a=\sqrt{\dfrac{2z-4(-156561274+37635273\sqrt{2}-30656666\sqrt{3}+9648410\sqrt{6})}{988438611}}$ （$\fallingdotseq 0.335266$）

ただし，$z=\sqrt[3]{e+5930631666\sqrt{f}}+\sqrt[3]{e-5930631666\sqrt{f}}$，

$e=144178258771658491200385 1-225089952658618737186881 55\sqrt{2}+134263492859109386374549 8\sqrt{3}-$

$119108506410498761147691 90\sqrt{6}$，

$f=3(176992534925914585707731 80684229+143207252084248229575912 64560662\sqrt{2}+$

$100368135414003994457585326 75191\sqrt{3}+809409980918182395634549 7677592\sqrt{6})$

(25) $a=\dfrac{2\sqrt{3}}{\sqrt{\sqrt[3]{e+f}+\sqrt[3]{e-f}+23-2\sqrt{2}+4\sqrt{3}+2\sqrt{6}+2(\sqrt{2}+\sqrt{6})g}}$ （$a\fallingdotseq 0.330562$）

だたし，$e=2\{4375+3280\sqrt{2}+510\sqrt{3}+1566\sqrt{6}+(3626+2351\sqrt{2}+328\sqrt{3}+1271\sqrt{6})g\}$，

$f=6\sqrt{3965553+1905972\sqrt{2}-855864\sqrt{3}-779952\sqrt{6}+6(283206+113645\sqrt{2}-83720\sqrt{3}-8403\sqrt{6})g}$，

$g=\sqrt{-2-\sqrt{2}+\sqrt{3}+\sqrt{6}}$

(26) $a=\dfrac{-2+2\sqrt{26}}{25}$ （$a\fallingdotseq 0.327922$）

(27) $a=\dfrac{-2+\sqrt{3}+\sqrt{19-10\sqrt{3}}}{3}$ （$a\fallingdotseq 0.342668$）

(28) $a=\dfrac{-307+141\sqrt{2}-103\sqrt{3}+99\sqrt{6}+\sqrt{549450-296934\sqrt{2}+228052\sqrt{3}-194574\sqrt{6}}}{529}$ （$\fallingdotseq 0.331553$）

(29) $a=\dfrac{-307+141\sqrt{2}-103\sqrt{3}+99\sqrt{6}+\sqrt{549450-296934\sqrt{2}+228052\sqrt{3}-194574\sqrt{6}}}{529}$ （$\fallingdotseq 0.331553$）

補足 (28) と同じ答え

Ⅱ

(1) $a=\dfrac{-6+5\sqrt{2}+4\sqrt{3}-3\sqrt{6}}{2}$ （$\fallingdotseq 0.325401$）

(2) $a=\dfrac{2}{3}\sqrt{\dfrac{1179-630\sqrt{2}-558\sqrt{3}+318\sqrt{6}}{433}}$ （$\fallingdotseq 0.321178$）

(3) $a=\dfrac{-10+21\sqrt{2}-8\sqrt{3}+3\sqrt{6}}{46}$ （$\fallingdotseq 0.286751$）

(4) $a=2\sqrt{\dfrac{6131-2454\sqrt{2}+1702\sqrt{3}-1830\sqrt{6}}{55897}}$ （$\fallingdotseq 0.283848$）

(5) $a=\dfrac{2}{3}\sqrt{\dfrac{1179-630\sqrt{2}-558\sqrt{3}+318\sqrt{6}}{433}}$ （$\fallingdotseq 0.321178$）

補足 (2) と同じ答え

(6) $a=\dfrac{2}{3}\sqrt{\dfrac{1179-630\sqrt{2}-558\sqrt{3}+318\sqrt{6}}{433}}$ （$\fallingdotseq 0.321178$）

補足 (2) と同じ答え

(7) $a=\dfrac{-10+21\sqrt{2}-8\sqrt{3}+3\sqrt{6}}{46}$ （$\fallingdotseq 0.286751$）

補足 (3) と同じ答え

(8) $a=\dfrac{-10+21\sqrt{2}-8\sqrt{3}+3\sqrt{6}}{46}$ （$\fallingdotseq 0.286751$）

補足 (3) と同じ答え

(9) $a=\dfrac{2(14-2\sqrt{2}+9\sqrt{3}-8\sqrt{6})}{47}$ （$\fallingdotseq 0.304856$）

(10) $a=\sqrt{\dfrac{4-2\sqrt{2}+\sqrt{3}-\sqrt{6}}{5}}$ （$\fallingdotseq 0.301375$）

(11) $a=\dfrac{2(14-2\sqrt{2}+9\sqrt{3}-8\sqrt{6})}{47}$ （$\fallingdotseq 0.304856$）

補足 (9) と同じ答え

(12) $a=\dfrac{2(14-2\sqrt{2}+9\sqrt{3}-8\sqrt{6})}{47}$ $(\fallingdotseq 0.304856)$

補足 (9) と同じ答え

(13) $a=\dfrac{5-\sqrt{3}}{11}$ $(\fallingdotseq 0.297086)$

(14) $a=2\sqrt{\dfrac{29-10\sqrt{3}}{541}}$ $(\fallingdotseq 0.293862)$

(15) $a=\dfrac{5-\sqrt{3}}{11}$ $(\fallingdotseq 0.297086)$

補足 (13) と同じ答え

(16) $a=\dfrac{5-\sqrt{3}}{11}$ $(\fallingdotseq 0.297086)$

補足 (13) と同じ答え

(17) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$ $(\fallingdotseq 0.343724)$

(18) $a=\dfrac{2(4-\sqrt{3})}{13}$ $(\fallingdotseq 0.348915)$

(19) $a=\dfrac{2(14-2\sqrt{2}+9\sqrt{3}-8\sqrt{6})}{47}$ $(\fallingdotseq 0.304856)$

補足 (9) と同じ答え

(20) $a=\dfrac{2(14-2\sqrt{2}+9\sqrt{3}-8\sqrt{6})}{47}$ $(\fallingdotseq 0.304856)$

補足 (9) と同じ答え

(21) $a=2\sqrt{\dfrac{29-10\sqrt{3}}{541}}$ $(\fallingdotseq 0.293862)$

補足 (14) と同じ答え

(22) $a=\dfrac{5-\sqrt{3}}{11}$ $(\fallingdotseq 0.297086)$

補足 (13) と同じ答え

(23) $a=\dfrac{3+4\sqrt{2}+7\sqrt{3}-6\sqrt{6}}{23}$ $(\fallingdotseq 0.264534)$

(24) $a=\dfrac{3+4\sqrt{2}+7\sqrt{3}-6\sqrt{6}}{23}$ $(\fallingdotseq 0.264534)$

補足 (23) と同じ答え

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